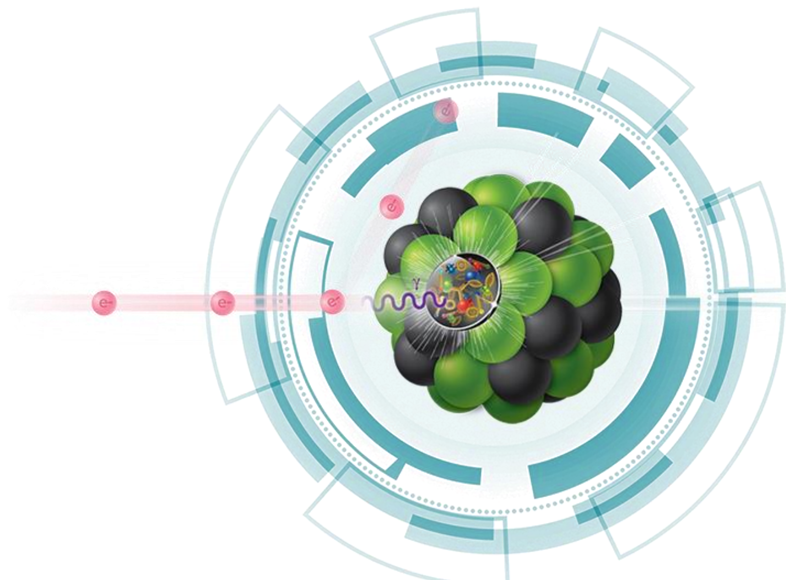

ÉCOLE POLYTECHNIQUE

CPHT Centre de Physique Théorique

cnrs

# Tomography at the femtometer scale

to reveal hadron structure



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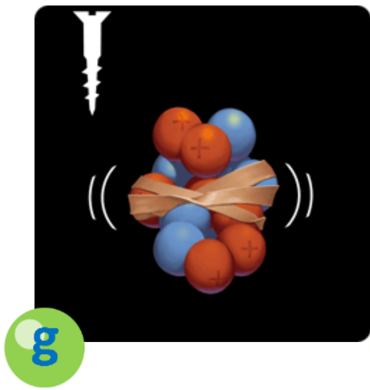
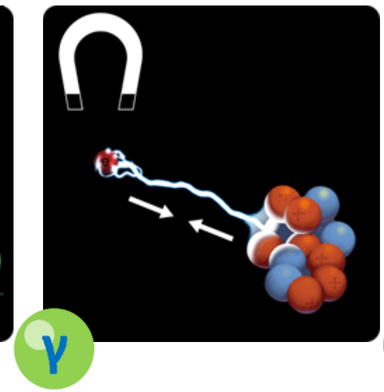
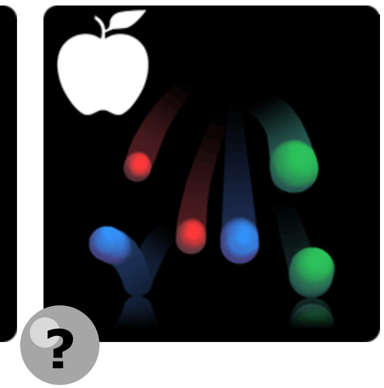
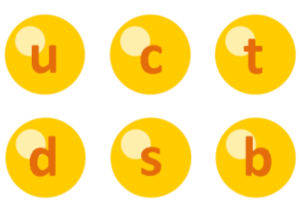
INSTITUT POLYTECHNIQUE DE PARIS

# Our world according to Particle Physics

## Interactions

| Strong | Weak | Electro magnetic | Gravitational |
|---|---|---|---|
|   |   |   |   |

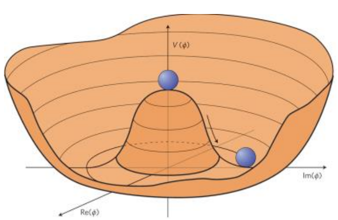
## Matter




Quarks


Leptons

## Particle mass

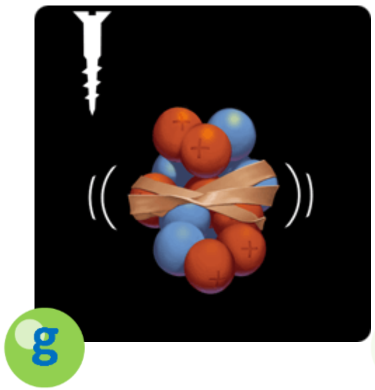



Higgs boson

# Quantum Chromodynamics (QCD)

## Interactions

**Strong**




Weak



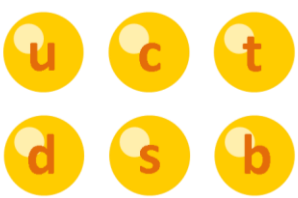

Electro
magnetic




Gravitational




## Matter




**Quarks**




Leptons

## Particle mass



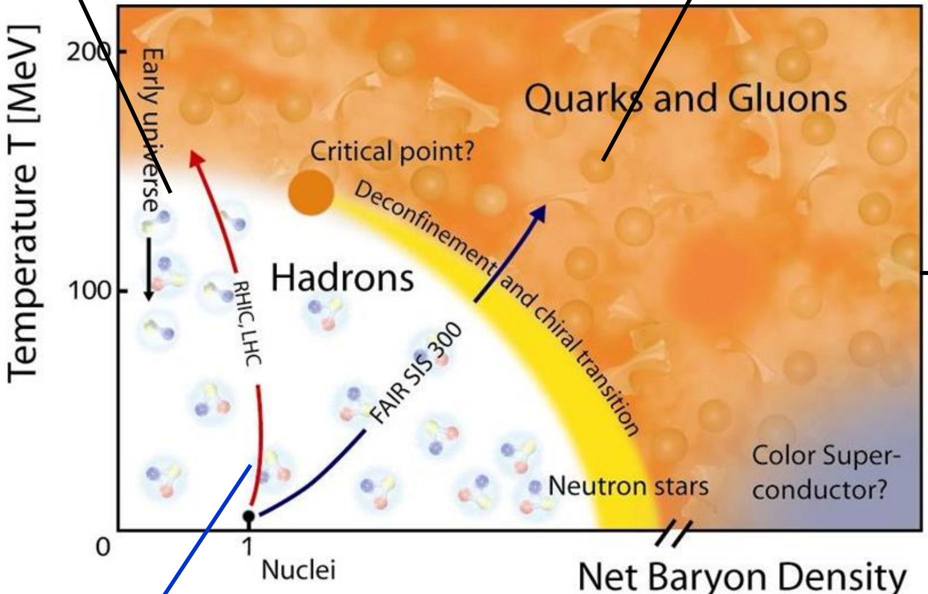



Higgs boson

# Questions addressed @CPHT

**Color glass condensate?**

**Quark-gluon plasma?**




**QCD phase diagram?**

**Hadron structure?**

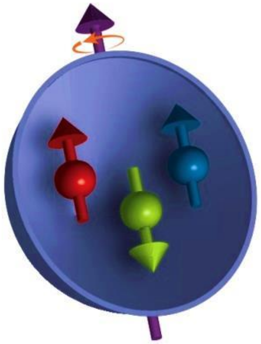
# Nucleon structure

## Non-relativistic picture

**dominated by constituents**

**Until ~ 1980**

Spectroscopy



**Mass**

$$M_N \sim \sum_Q M_Q + E_{\text{binding}}$$

**~ 102 %** **~ - 2 %**

**Spin**

$$J_z^N \sim \sum_Q S_z^Q$$

**~ 100 %**

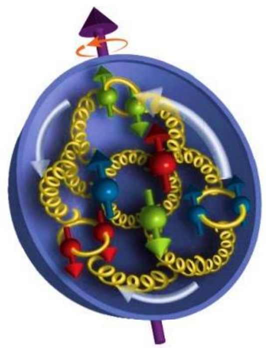
# Nucleon structure

## Relativistic picture

**dominated by dynamics**

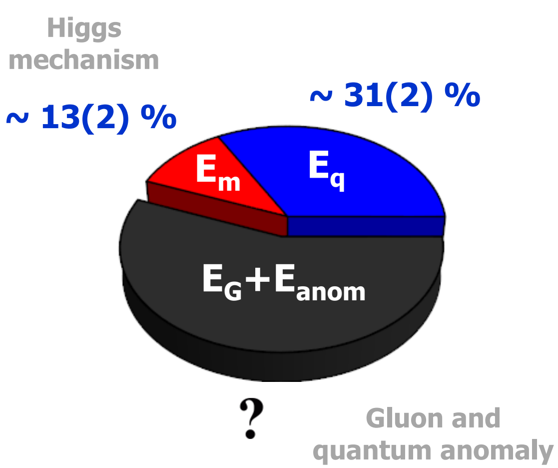
**Now**

High-energy scattering

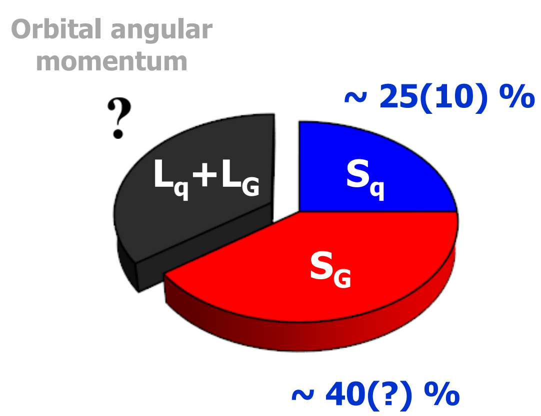



- Quantum
- Relativistic
- Non-perturbative
- Non-abelian gauge symmetry
- Confined constituents

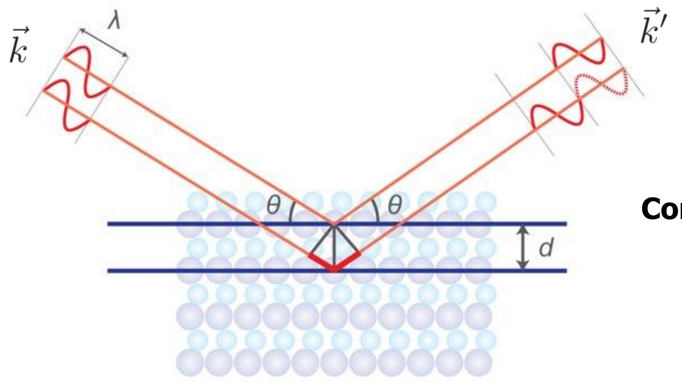
## Mass




## Spin



# Elastic scattering

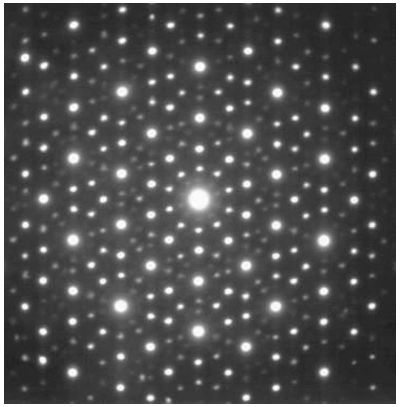



**Crystal**

**Constructive interference**
**(Bragg's law)**

$$2d\,\sin\theta = n\lambda$$

**Diffraction pattern**



$\propto |A_{\text{scatt}}|^2$

## Scattered amplitude

$$A_{\text{scatt}} \propto F(\vec{q}) = \int \mathrm{d}^3 r\, e^{i\vec{q}\cdot\vec{r}}\, \rho(\vec{r}) \qquad \vec{q} = \vec{k} - \vec{k}'$$

**Form factor** **Scatterer distribution**

## Reconstructed charge distribution

$$\rho(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}}\, F(\vec{q})$$

**Let's replace the crystal by a nucleon!**

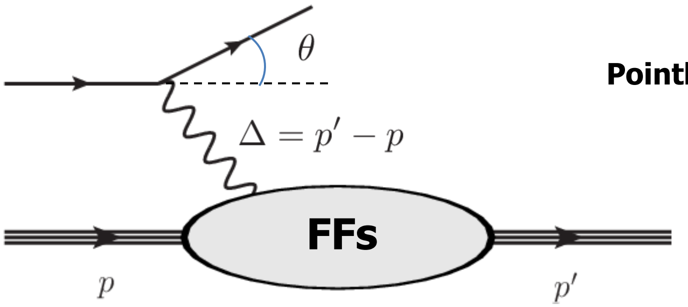
# Nucleon charge distribution

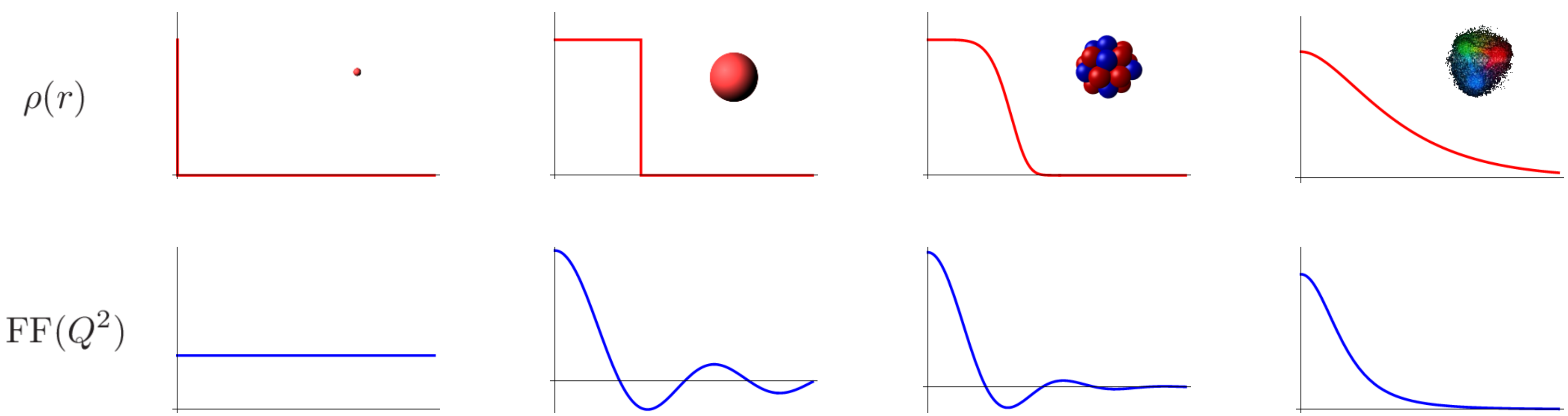
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left\{ F_1^2(Q^2) + \frac{Q^2}{4M^2}\left[F_2^2(Q^2) + 2[F_1(Q^2) + F_2(Q^2)]^2 \tan^2\frac{\theta}{2}\right]\right\}$$

**Pointlike** **Spatial structure**

$$Q^2 = -\Delta^2$$

$$\tau = \frac{Q^2}{4M^2}$$

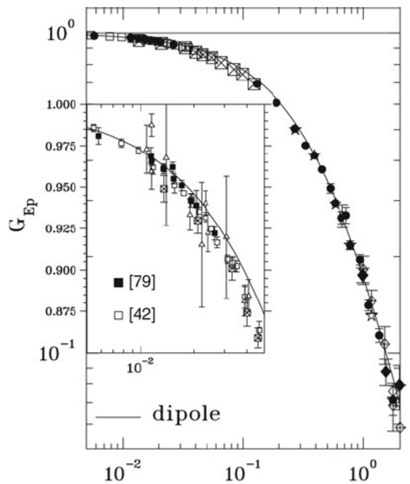


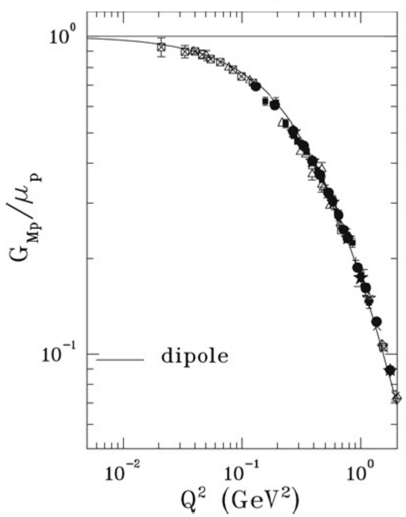



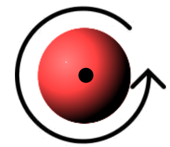
# 2D transverse charge distribution

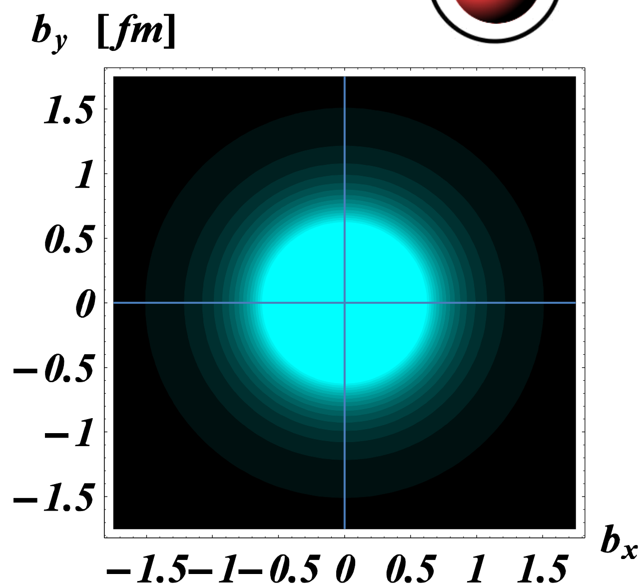
$$\rho(\vec{b}_\perp) = \int \frac{dQ}{2\pi} \, Q \, J_0(Qb_\perp) \, F_1(Q^2)$$

**Monopole**



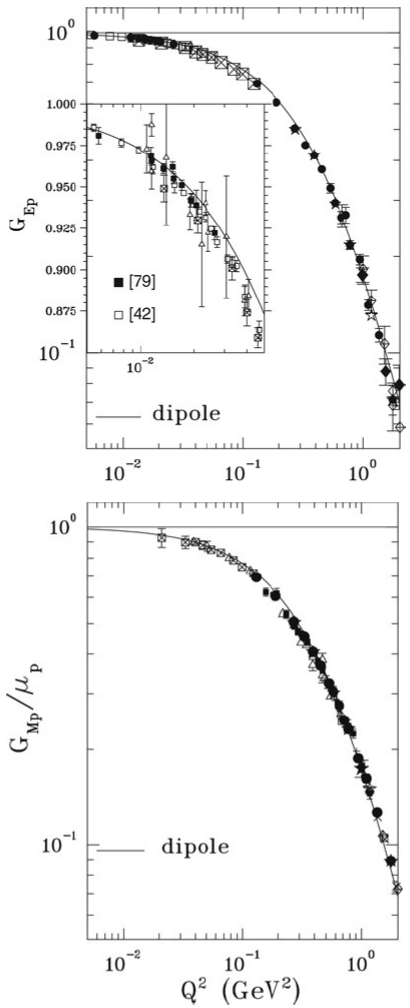



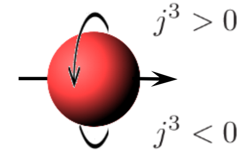


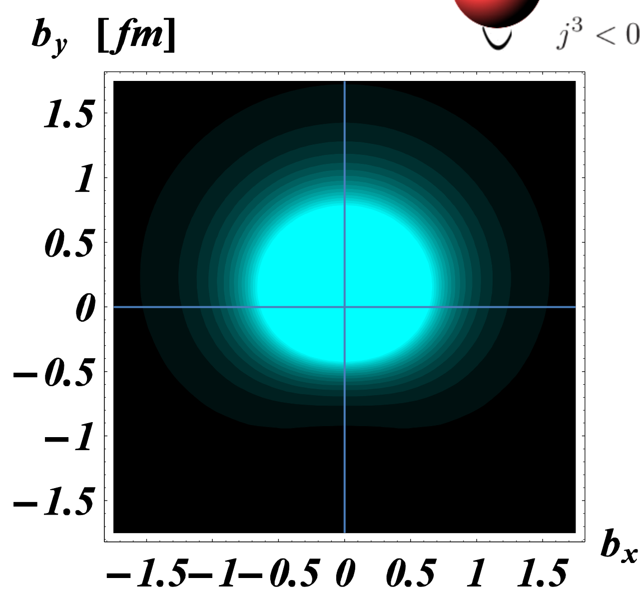
# 2D transverse charge distribution

$$\rho(\vec{b}_\perp) = \underbrace{\int \frac{dQ}{2\pi} Q\, J_0(Qb_\perp)\, F_1(Q^2)}_{\text{Monopole}} + \underbrace{\sin(\phi_b - \phi_S) \int \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(Qb_\perp)\, F_2(Q^2)}_{\text{Induced dipole}}$$

**Monopole** **Induced dipole**




**Clear signature of internal orbital angular momentum!**

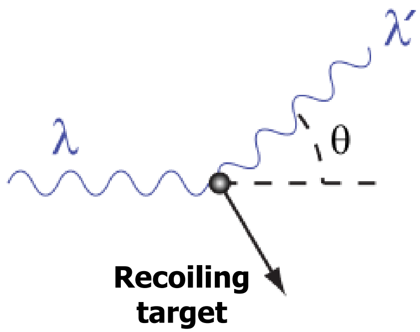
**Anomalous magnetic moment**

$$a = \frac{g-2}{2} = F_2(0)$$

$$a_e = 0.001\,159\,652\,180\,5$$
$$a_p = 1.792\,847\,351$$

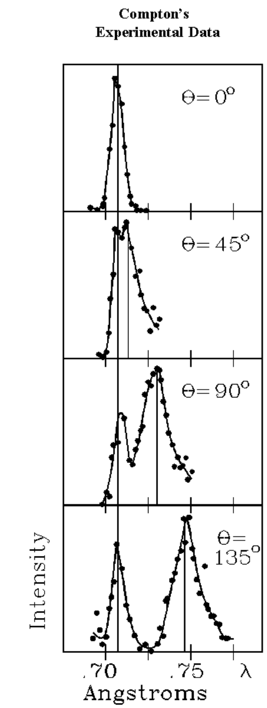
# Compton scattering

## Real Compton scattering

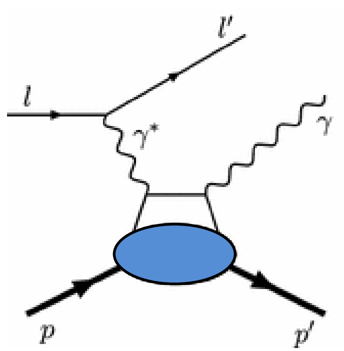



$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

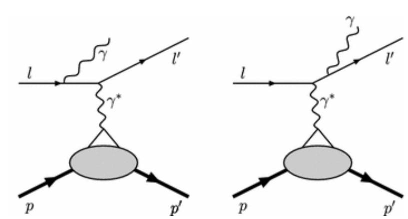
↑
**Compton wavelength**




## Deeply virtual Compton scattering (DVCS)




**interferes with**



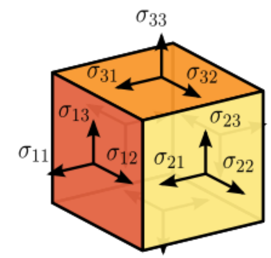

**Bethe-Heitler**

**2 close photons are similar to a graviton!**

$J=1$ $J=2$

# Energy-momentum tensor (EMT)

**Mass**, **spin** and **pressure** all encoded in




Energy density | Momentum density

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

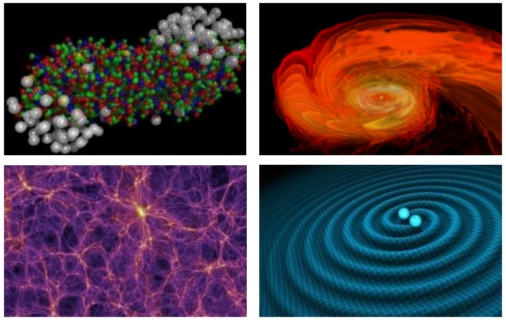
Shear stress

Normal stress (pressure)

Energy flux | Momentum flux

**Key concept for**

- **Nucleon mechanical properties**
- **Quark-gluon plasma**
- **Relativistic hydrodynamics**
- **Stellar structure and dynamics**
- **Cosmology**
- **Gravitational waves**
- **Modified theories of gravitation**
- **...**



# Anisotropic medium

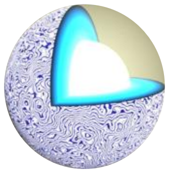
**Breit frame amplitude** $t = -\vec{\Delta}^2$

$$\frac{\langle \frac{\vec{\Delta}}{2}|T_i^{\mu\nu}(0)|-\frac{\vec{\Delta}}{2}\rangle}{2P^0} = M\left\{\eta^{\mu 0}\eta^{\nu 0}\left[A_i(t) + \frac{t}{4M^2}\,B_i(t)\right] + \eta^{\mu\nu}\left[\bar{C}_i(t) - \frac{t}{M^2}\,C_i(t)\right] + \frac{\Delta^\mu\Delta^\nu}{M^2}\,C_i(t)\right\}$$

**Gravitational form factor**

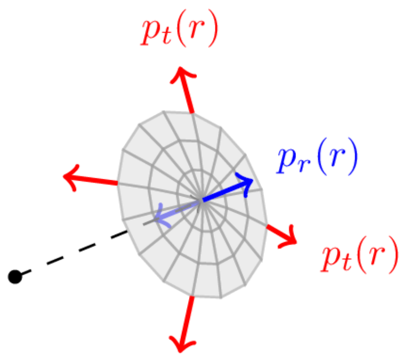
**Analogy with relativistic hydrodynamics** $r = |\vec{r}|$

**Like a neutron star!**



**Anisotropic fluid**

$$\Theta_i^{\mu\nu}(\vec{r}) = u^\mu u^\nu\left[\varepsilon_i(r) + p_{t,i}(r)\right] - \eta^{\mu\nu} p_{t,i}(r) + \frac{r^\mu r^\nu}{r^2}\left[p_{r,i}(r) - p_{t,i}(r)\right]$$




**Isotropic pressure**

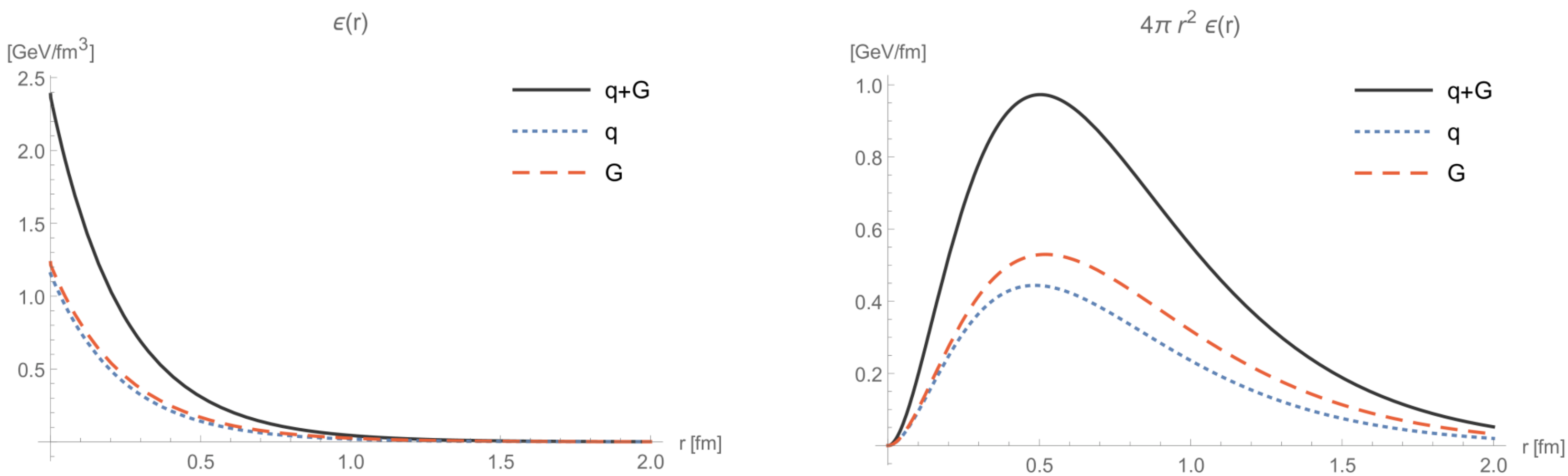
$$p_i(r) = \frac{p_{r,i}(r) + 2p_{t,i}(r)}{3}$$

**Pressure anisotropy**

$$s_i(r) = p_{r,i}(r) - p_{t,i}(r)$$

# Energy distribution

**Multipole model for the GFFs** $$F(t) = \frac{F(0)}{(1+t/\Lambda^2)^n}$$

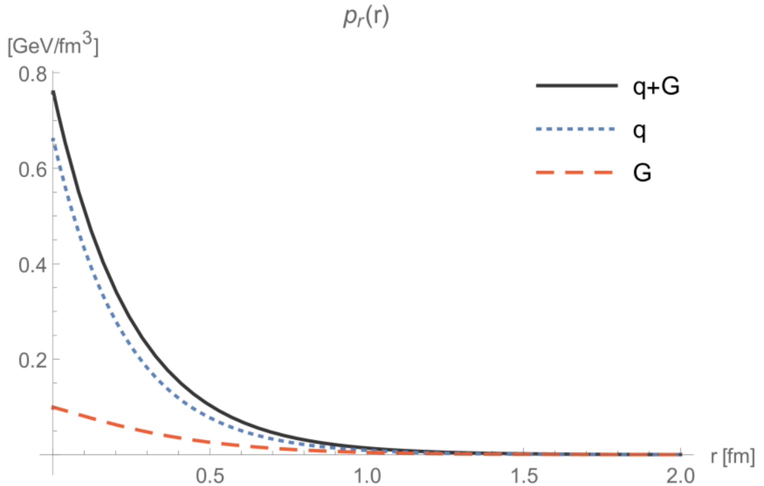


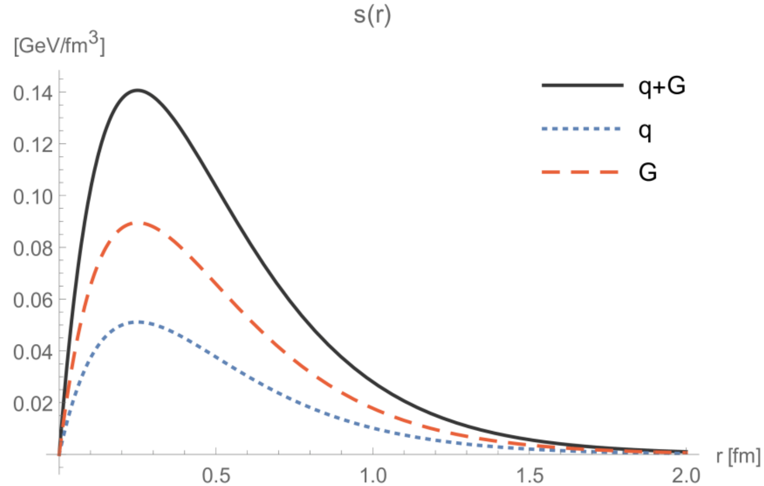

**Mass radius** $\sqrt{\langle r^2\rangle_M} = 0.91\,\mathrm{fm}$

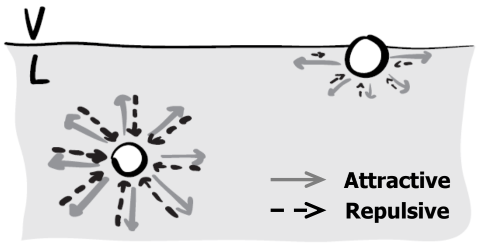
**Charge radius** $\sqrt{\langle r^2\rangle_Q} = 0.84 - 0.88\,\mathrm{fm}$

# Local equilibrium

$$\nabla^i T^{ij}(\vec{r}) = 0 \quad \Rightarrow \quad \frac{dp_r(r)}{dr} = -\frac{2s(r)}{r}$$




**Surface tension**

$$\gamma = \int dr\, s(r)$$

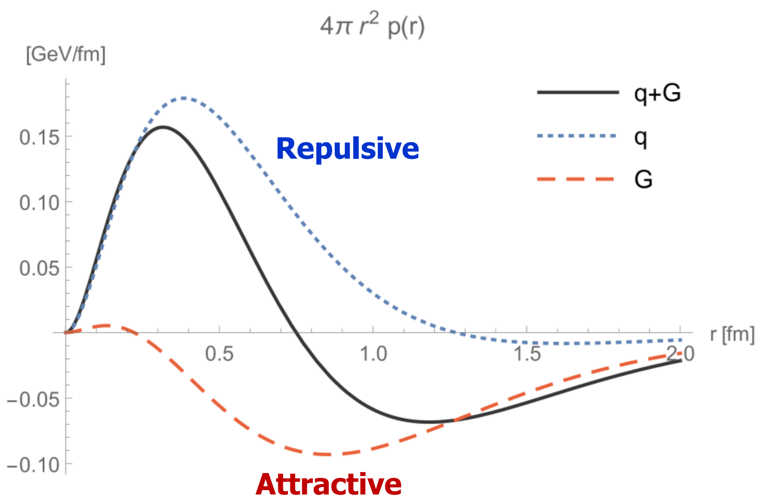
**Generalized Young-Laplace relation**

$$p(0) = 2\int_0^\infty dr\, \frac{s(r)}{r}$$

$$s(r) \approx \gamma\, \delta(r - R) \quad \Rightarrow \quad p(0) = \frac{2\gamma}{R}$$

# Global equilibrium

$$\int \mathrm{d}^3r\, r^k \nabla^i \mathcal{T}^{ij}(\vec{r}) = 0 \quad \Rightarrow \quad \int \mathrm{d}^3r\, \mathcal{T}^{kj}(\vec{r}) = 0$$




**von Laue relation**

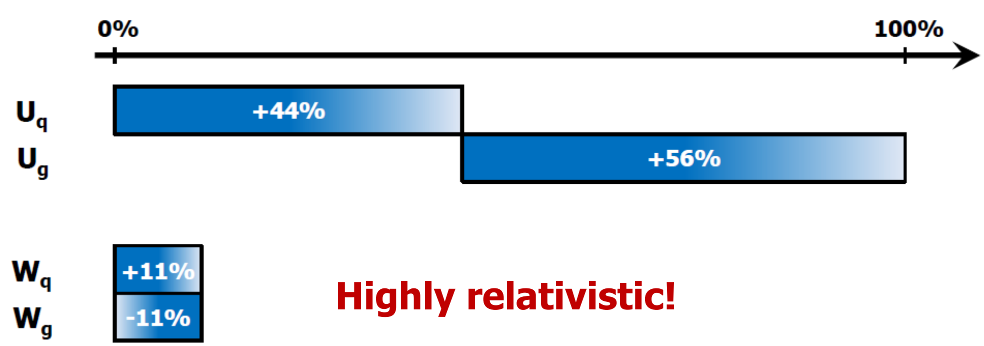
$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$

**Internal energy**

$$U_i = \int \mathrm{d}^3r\, \varepsilon_i(r)$$

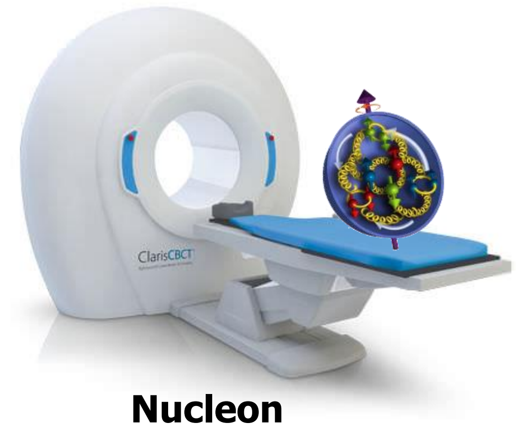
**Pressure-volume work**

$$W_i = \int \mathrm{d}^3r\, p_i(r)$$




**Highly relativistic!**

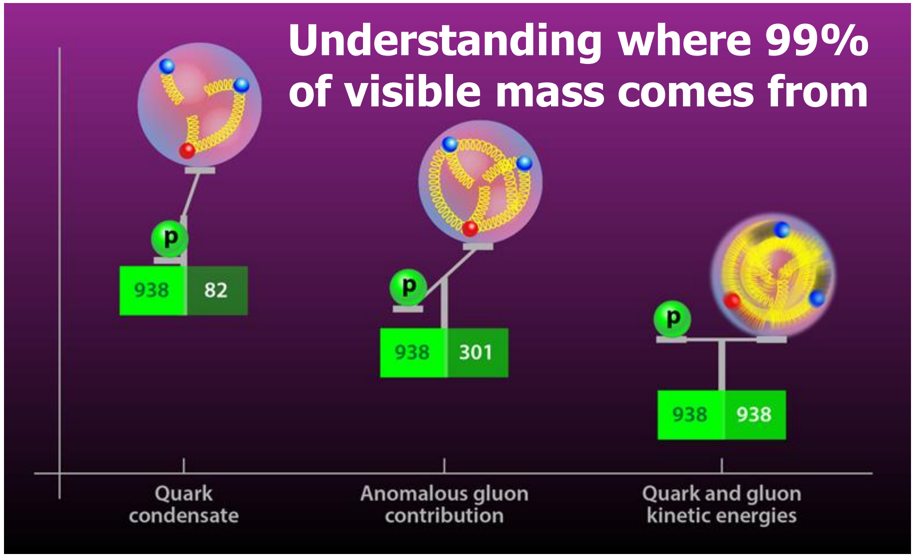
# Conclusion




**Nucleon**
**« tomography »**

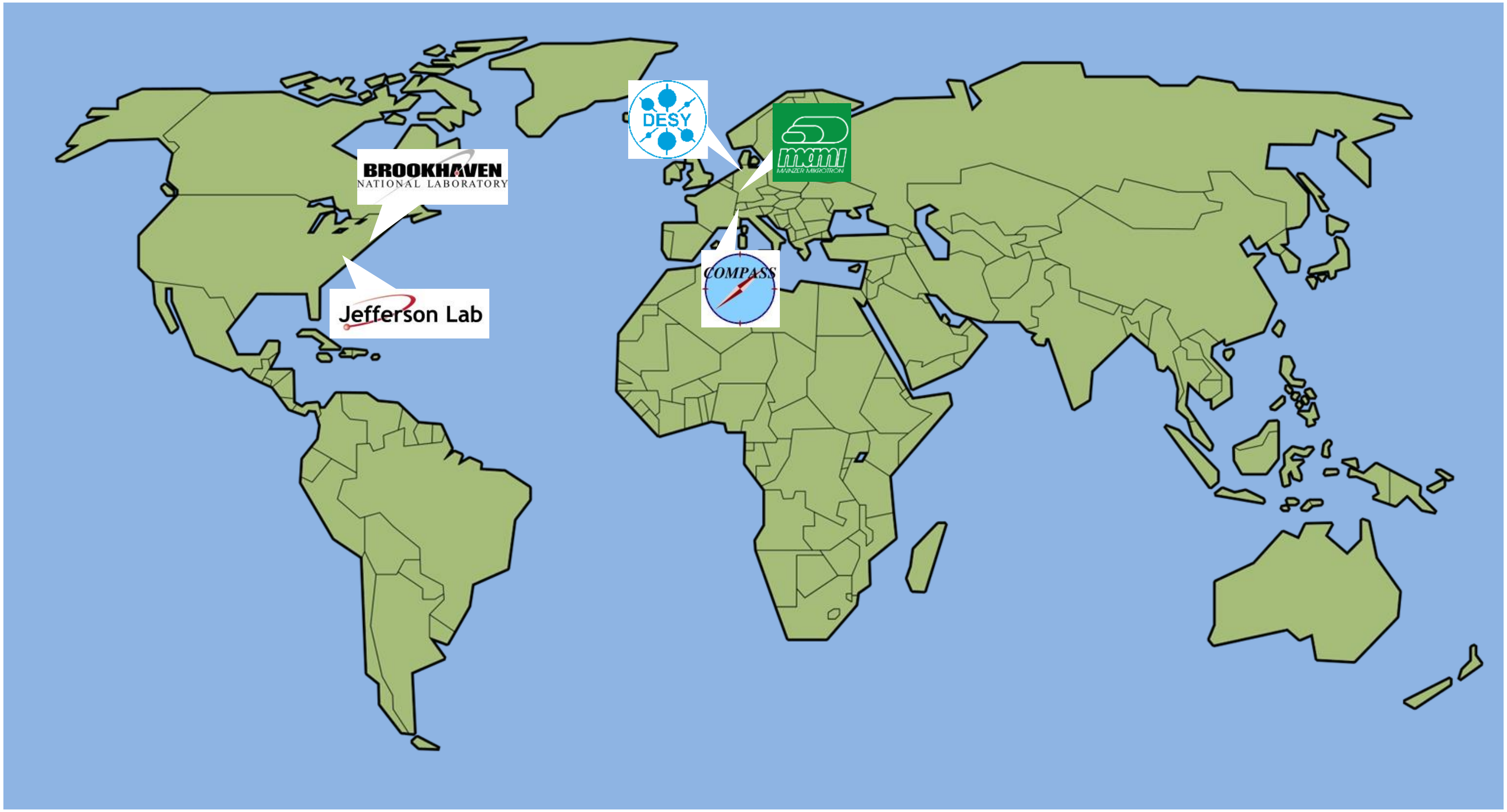
**High-energy electron-proton scattering is a key tool to unravel nucleon internal structure**

**Many exciting developments using the energy-momentum tensor!**



# Backup

# Experimental facilities



# Compact stars

White dwarfs, neutron stars, quark stars, strange stars, gravastars, black holes, …

**Highest energy densities and strongest gravitational fields!**

## Tests under extreme conditions

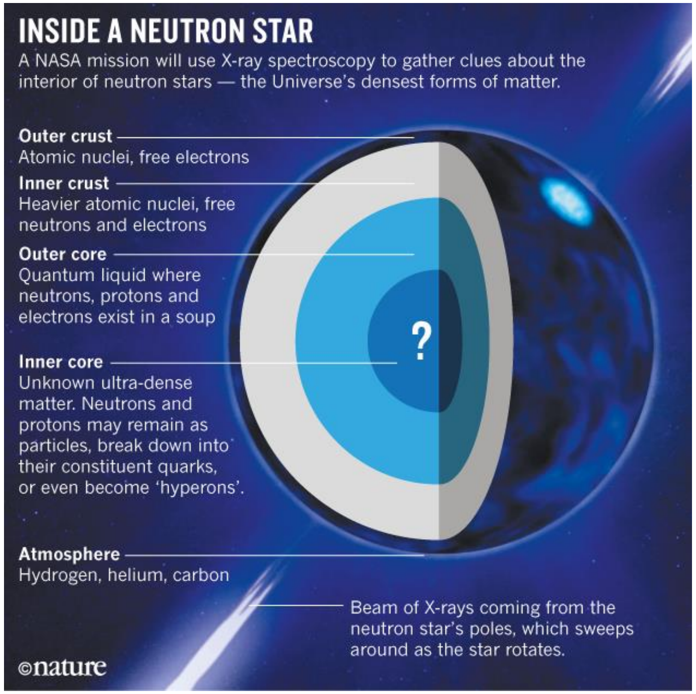
- Nuclear matter
- General relativity & alternatives

[Berti *et al.* (2015)]
[Lattimer, Prakash (2016)]

## EMT is likely anisotropic

- Relativistic nuclear interactions
- Mixture of fluids of different types
- Presence of superfluid
- Existence of solid core
- Phase transitions
- Presence of magnetic field
- Viscosity
- …

[Ruderman (1972)]
[Canuto (1974)]
[Bowers, Liang (1974)]
[Herrera, Santos (1997)]

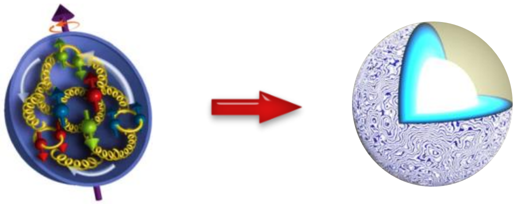



$$M \sim 1.4\, M_\odot \qquad M_\odot = 2 \times 10^{33}\,\mathrm{g}$$
$$R \sim 10\,\mathrm{km}$$
$$\rho \sim 3\,\rho_0 \qquad \rho_0 = 2.8 \times 10^{14}\,\mathrm{g/cm^3}$$
$$g \sim 2.4 \times 10^{12}\,\mathrm{m/s^2}$$

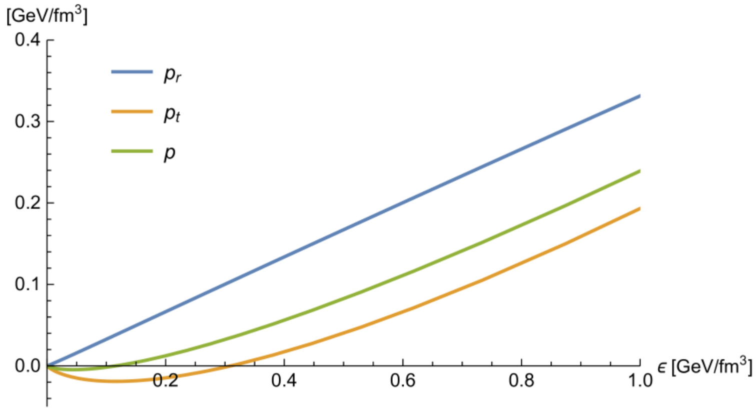
[Potekhin (2010)]

$$M_p \sim 1.67 \times 10^{-24}\,\mathrm{g}$$
$$R_p \sim 0.84\,\mathrm{fm}$$
$$\rho_p \sim 2.4\,\rho_0$$

# What can we learn?




**Equation of state**




[Annala *et al.* (2018)]