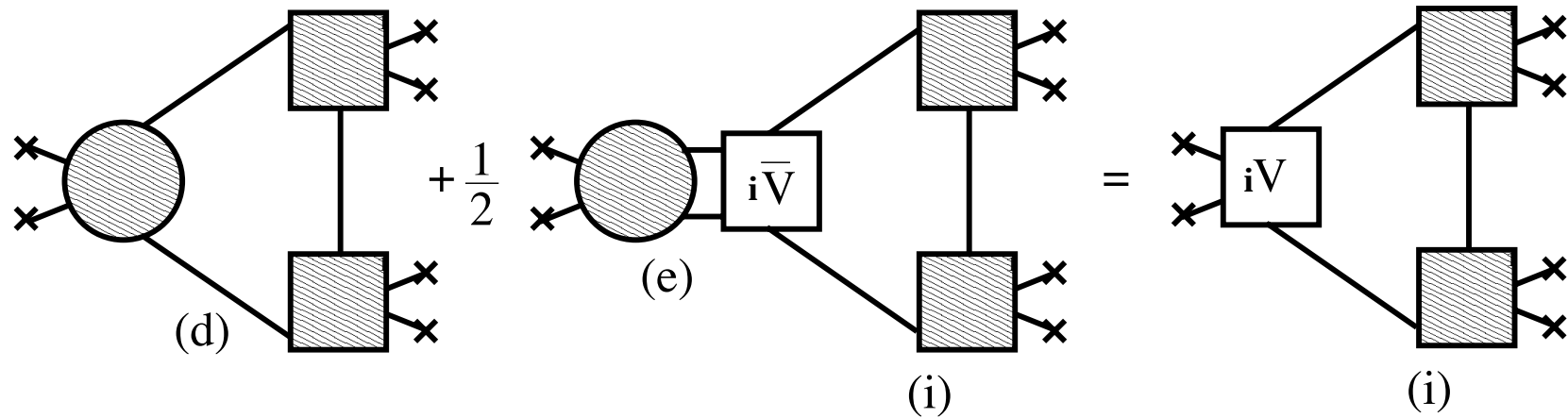
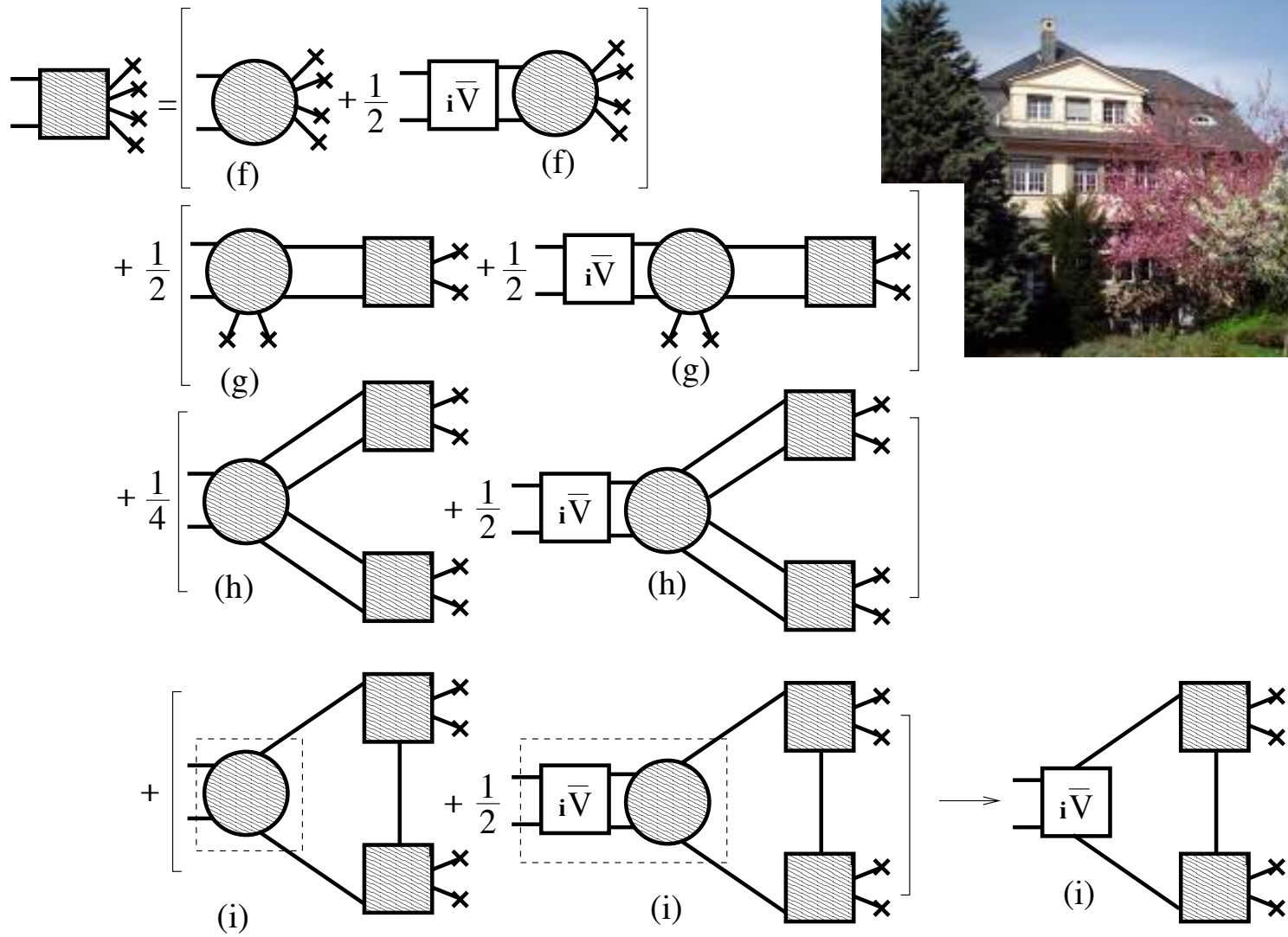


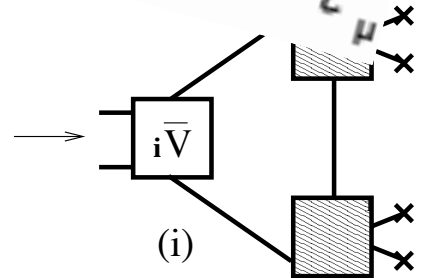
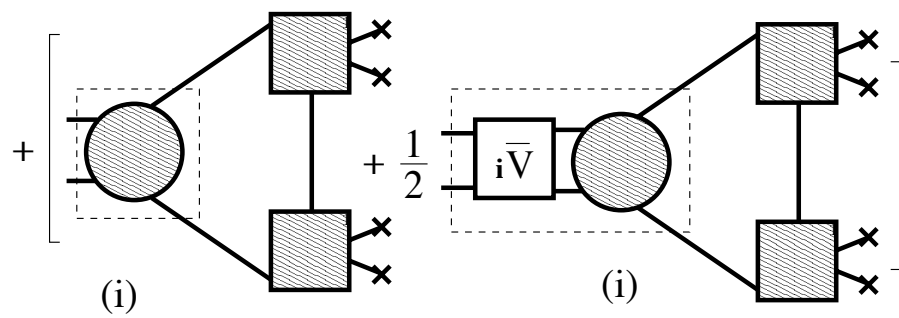
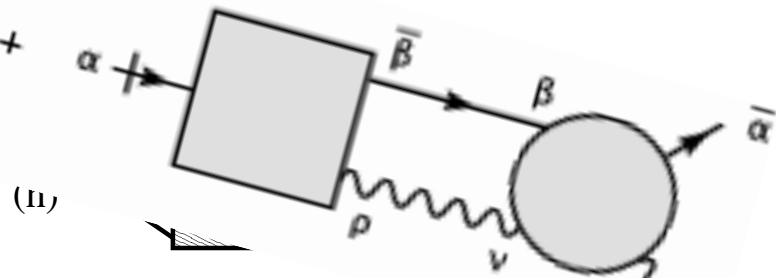
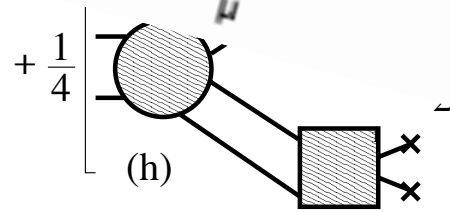
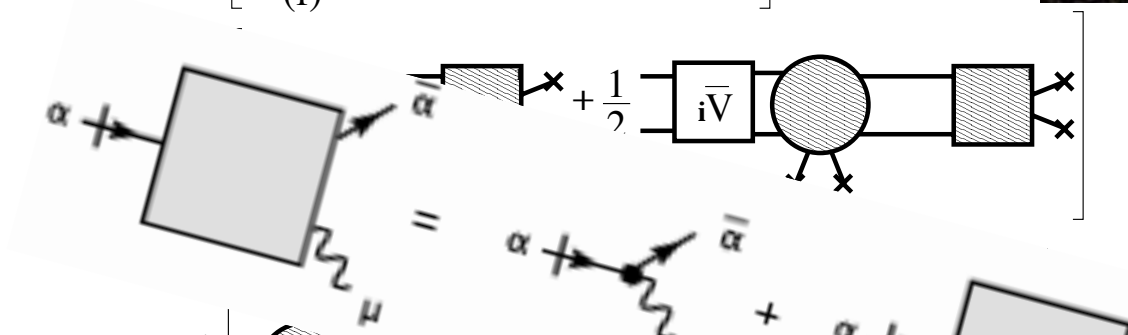
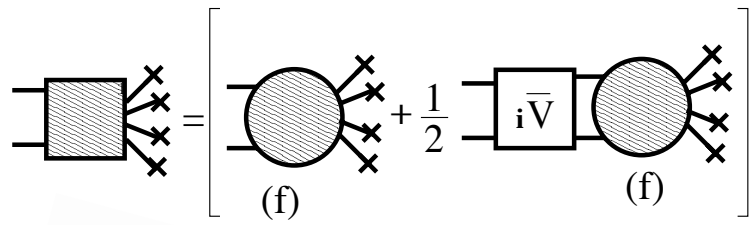
THE ~~MESS~~ OF THE GLUE  
A

Back in the days  
(Heidelberg, 2002...)



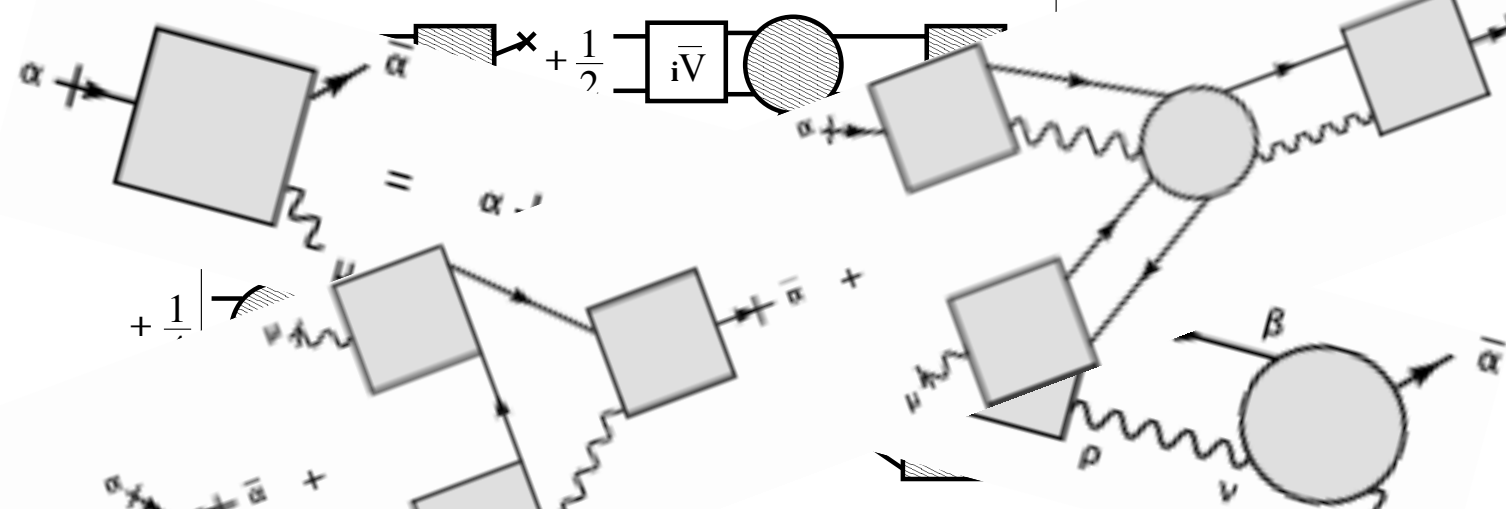
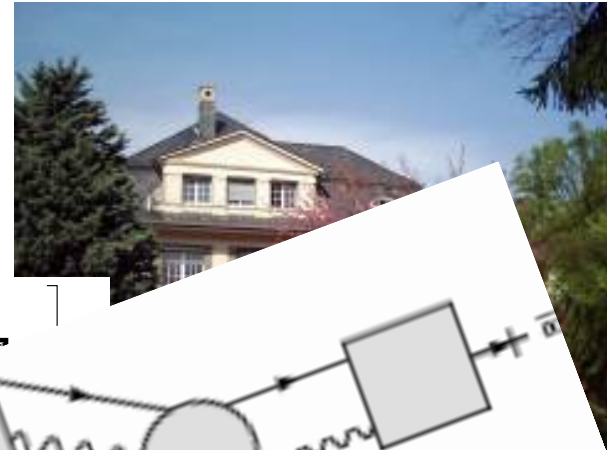


+ "perm."

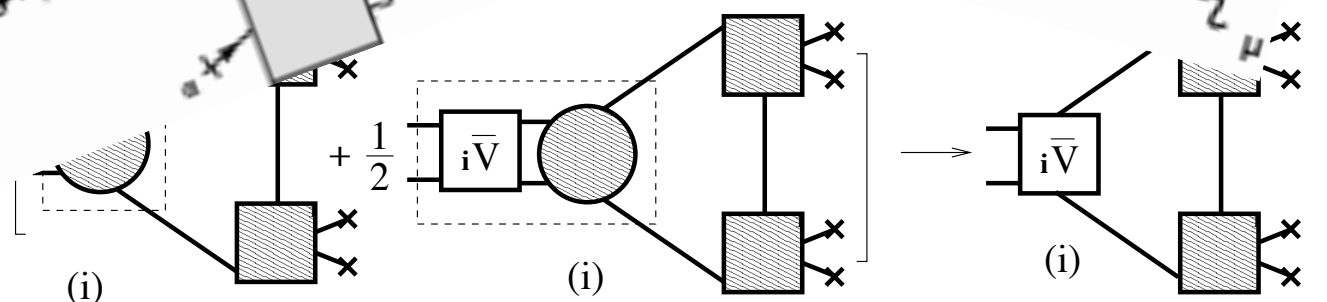


+ "perm."

$$\text{Diagram} = \left[ \text{Diagram (f)} + \frac{1}{2} \text{Diagram (f)} \right]$$

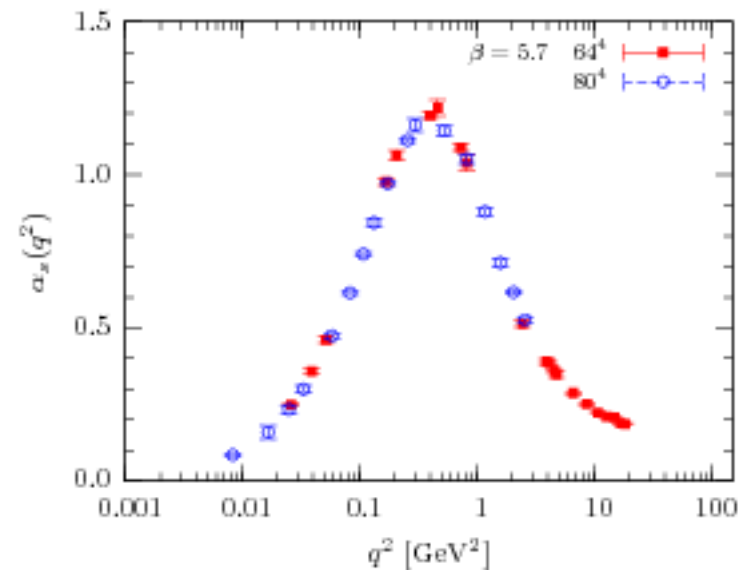
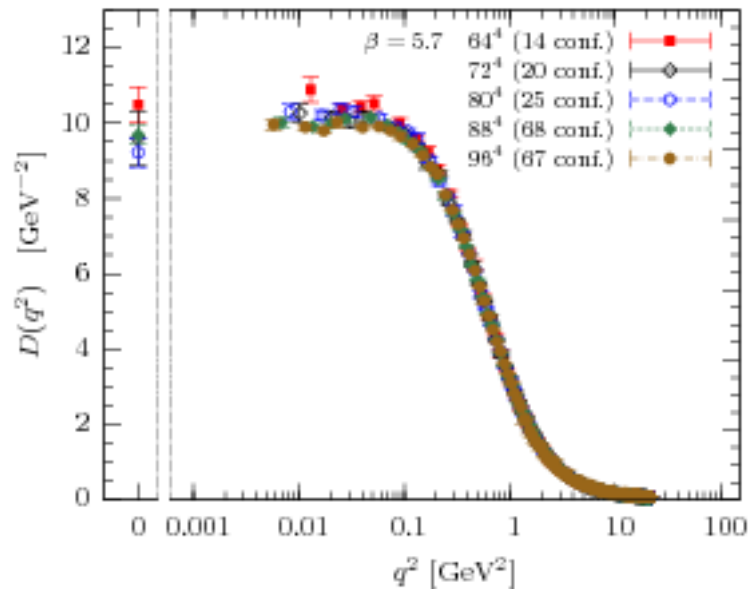
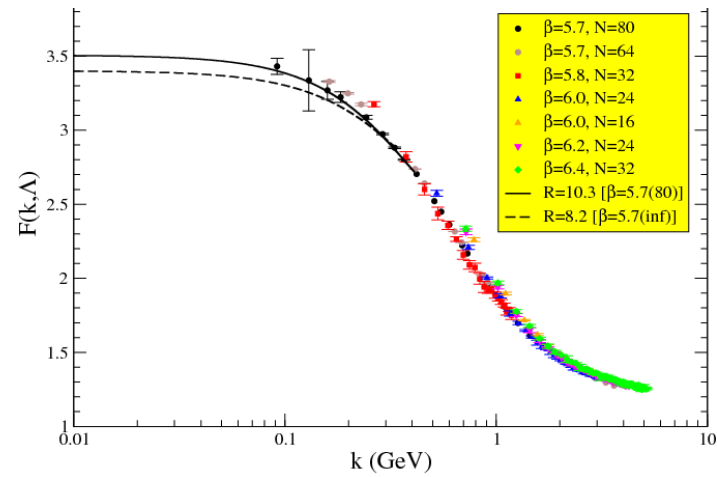


$$\Gamma_{R, \mu\alpha\alpha}^{(2,1)} =$$

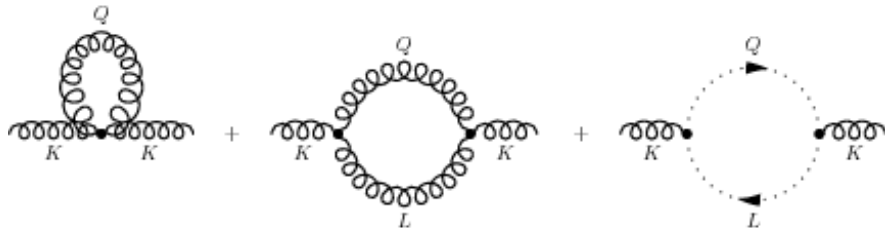


+ "perm."

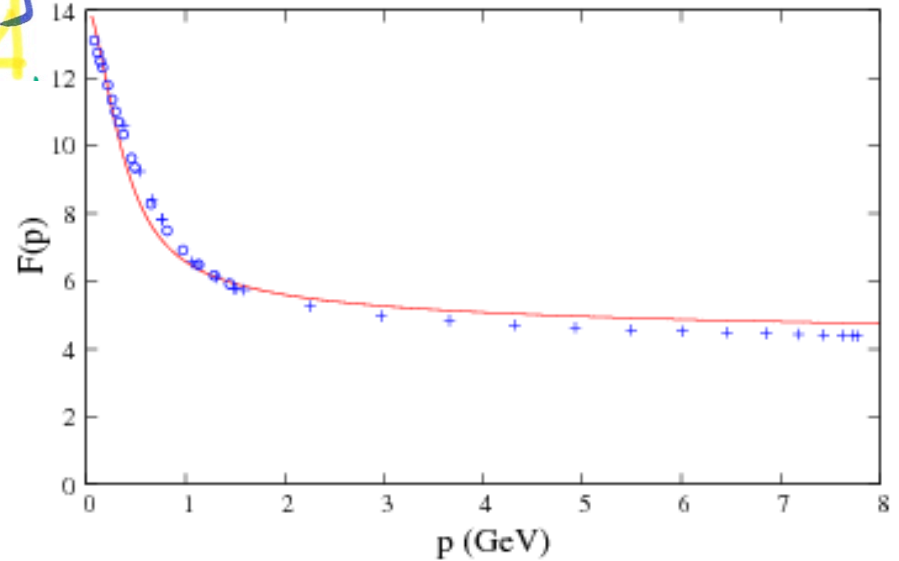
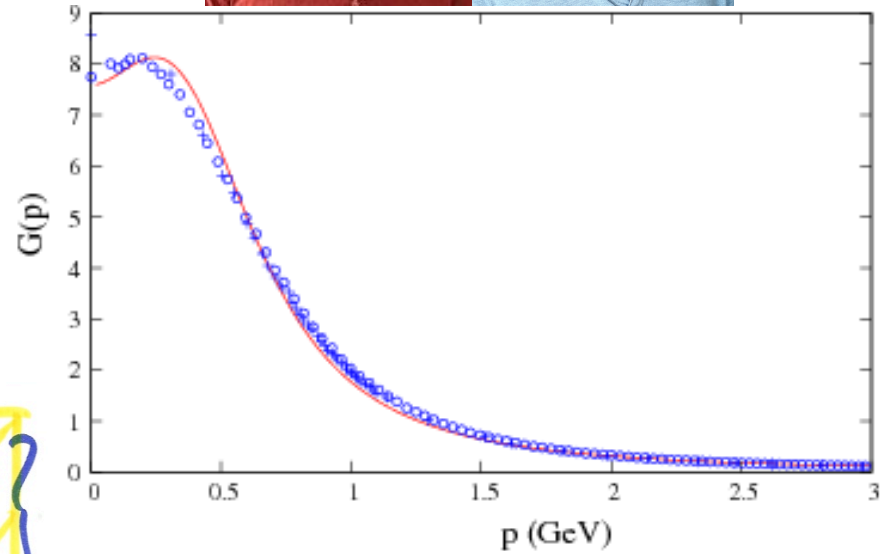
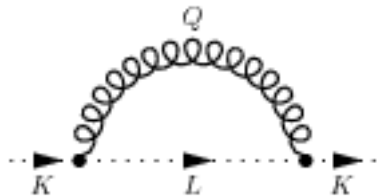
Meanwhile,  
on the lattice ...



# A BREAKTHROUGH (2010)



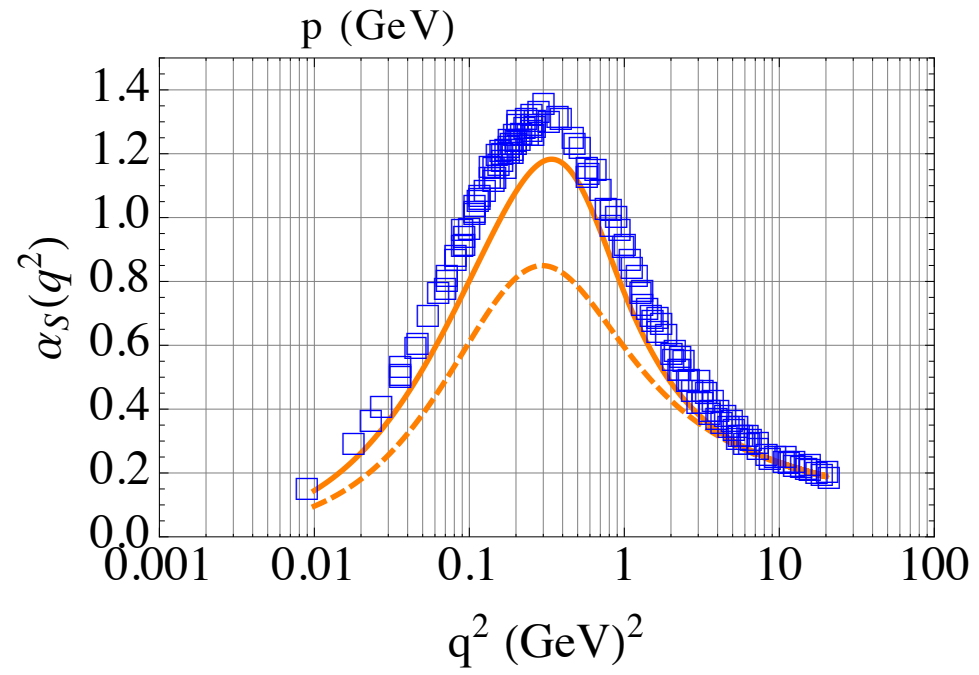
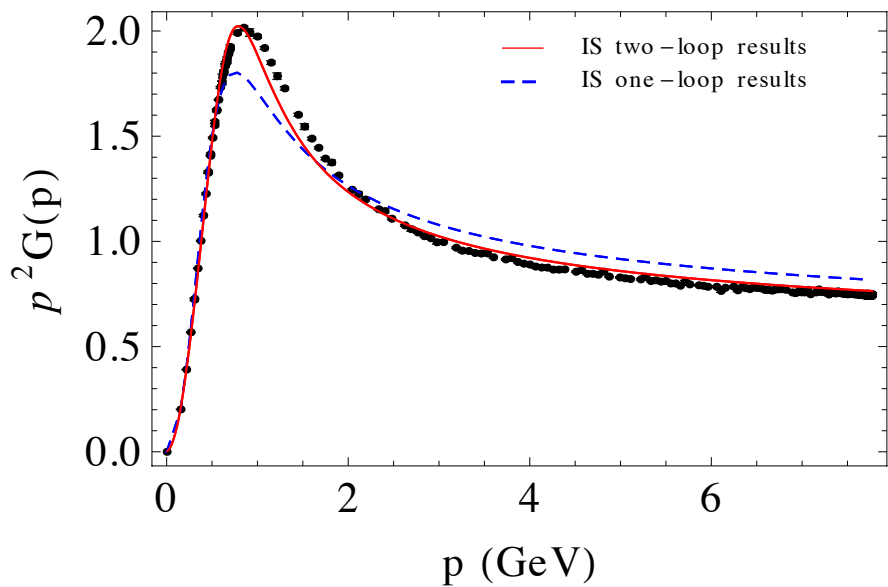
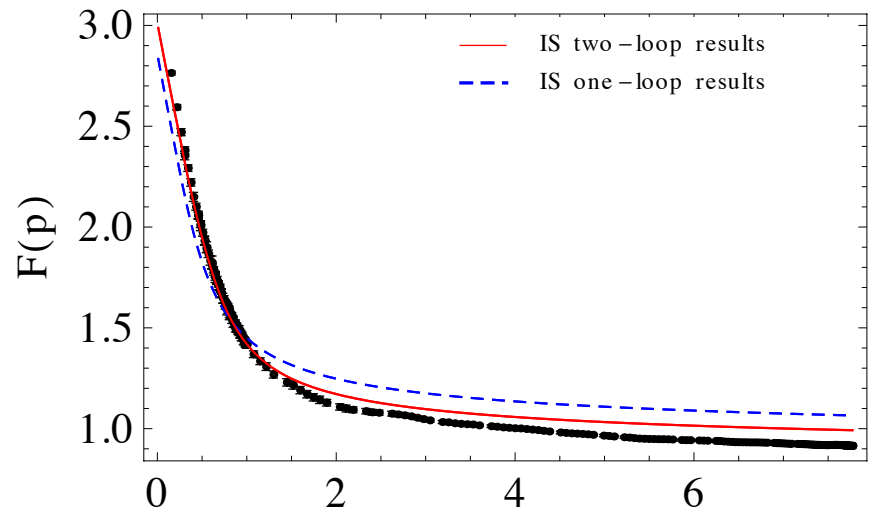
$$S = \int_x \left\{ \underbrace{\frac{F^2}{4} + i\hbar\partial A + \partial\bar{c}Dc}_{\text{Faddeev-Popov}} + \frac{m^2}{2}A^2 \right\}$$



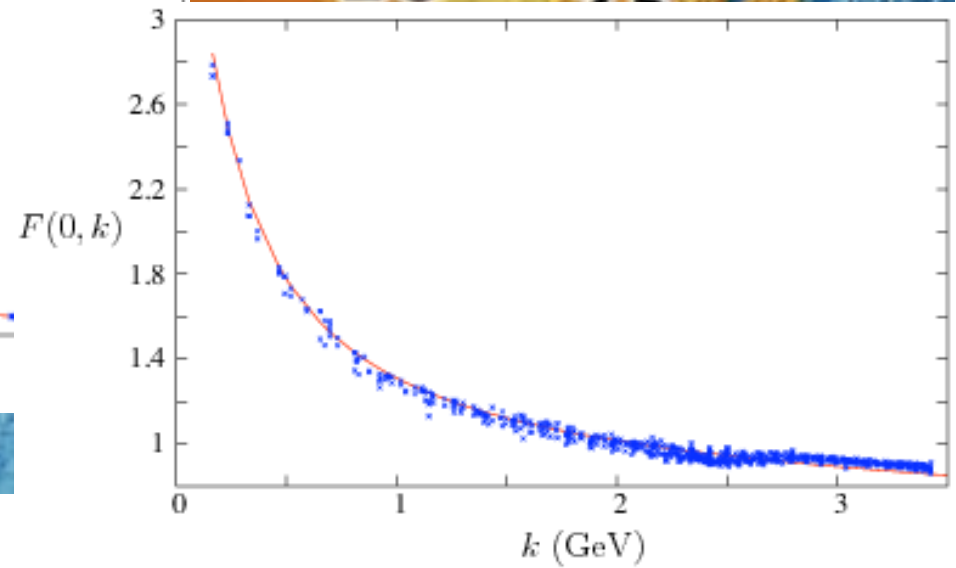
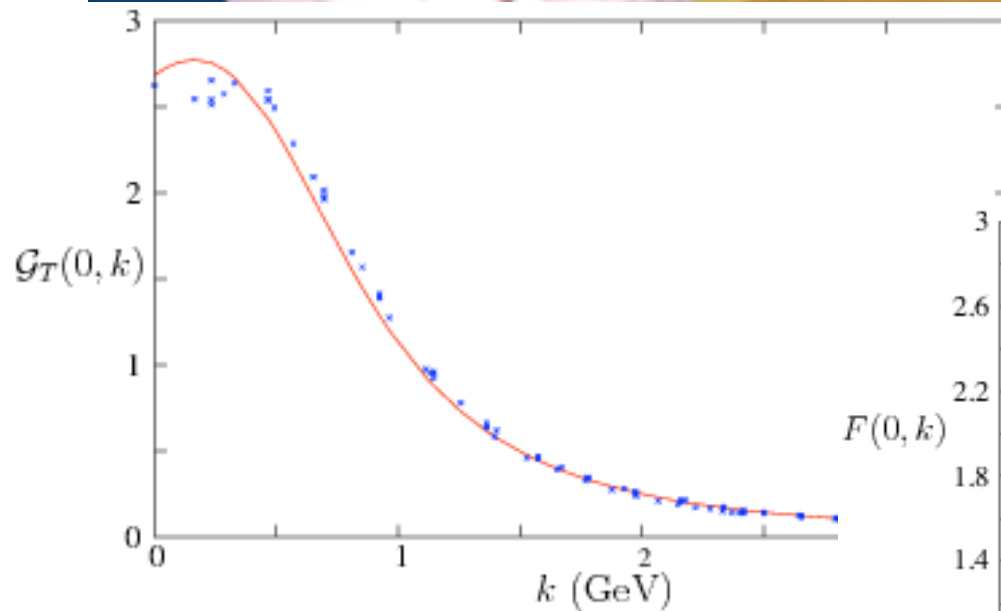
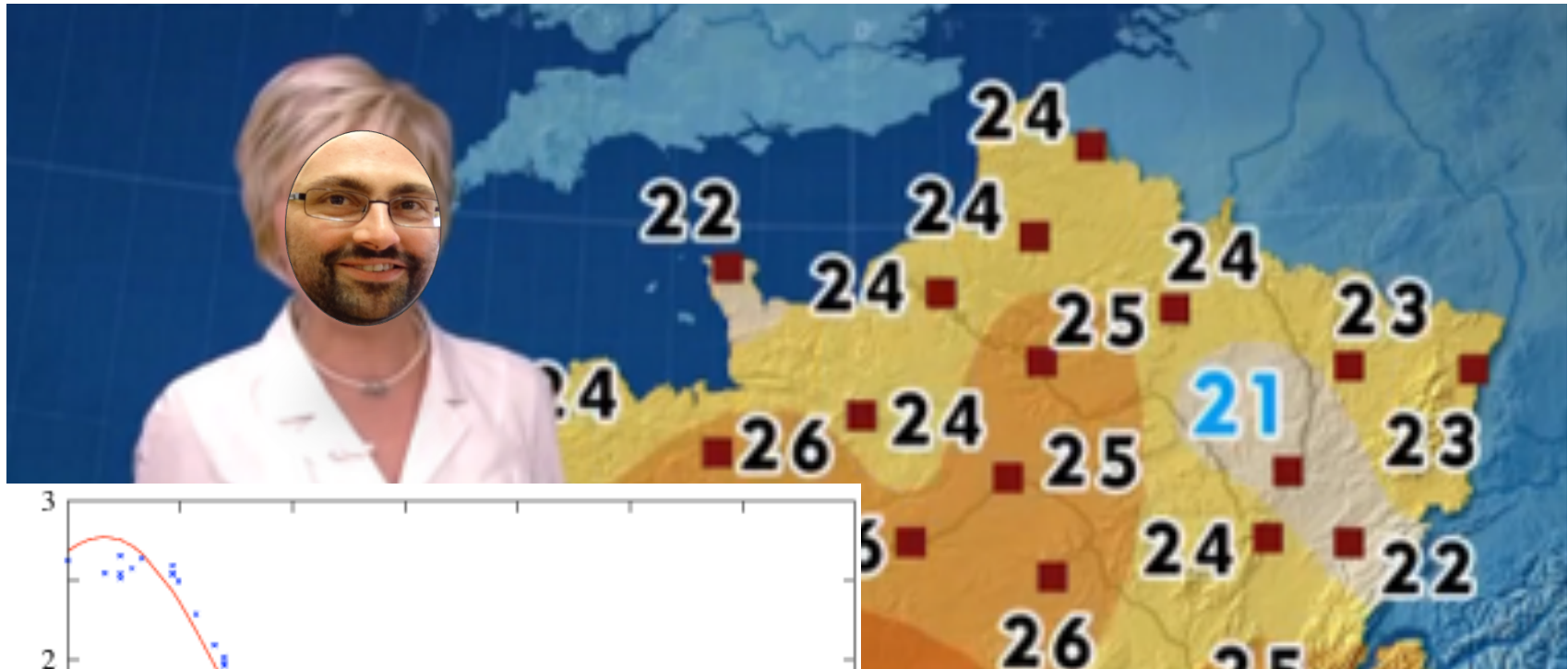




# Two Loop









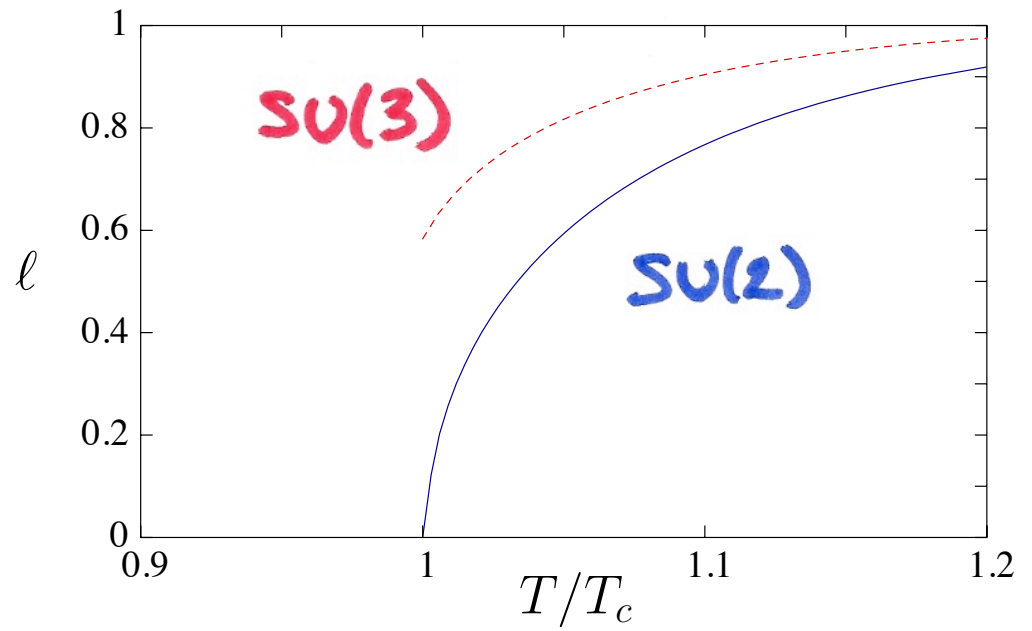
The Polyakov loop

$$L \sim e^{-F_9/T}$$



# The Polyakov loop

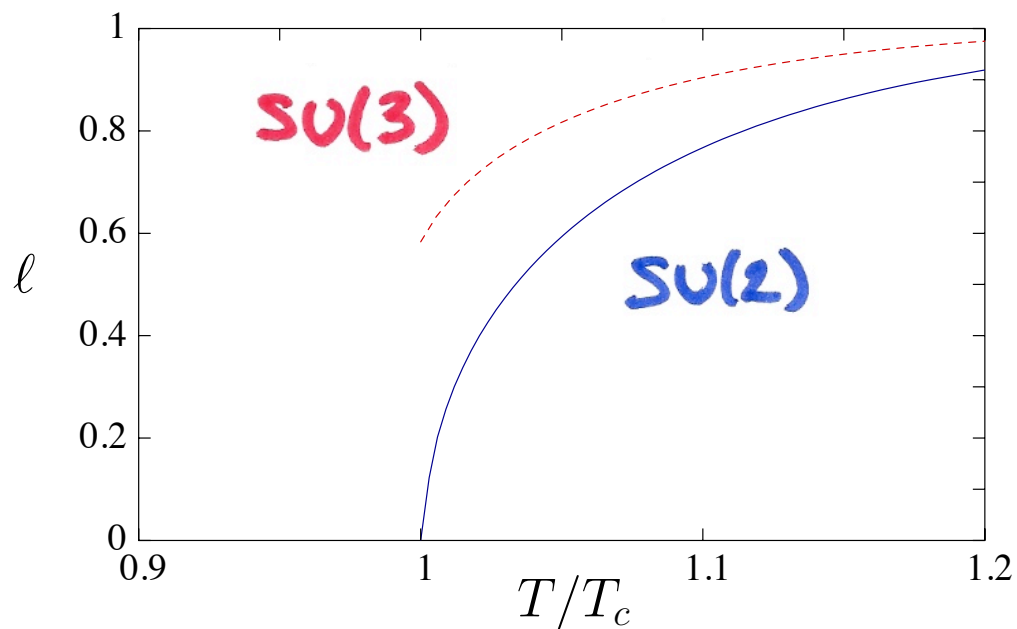
$$\ell \sim e^{-F_9/T}$$





# The Polyakov loop

$$l \sim e^{-F_9/T}$$



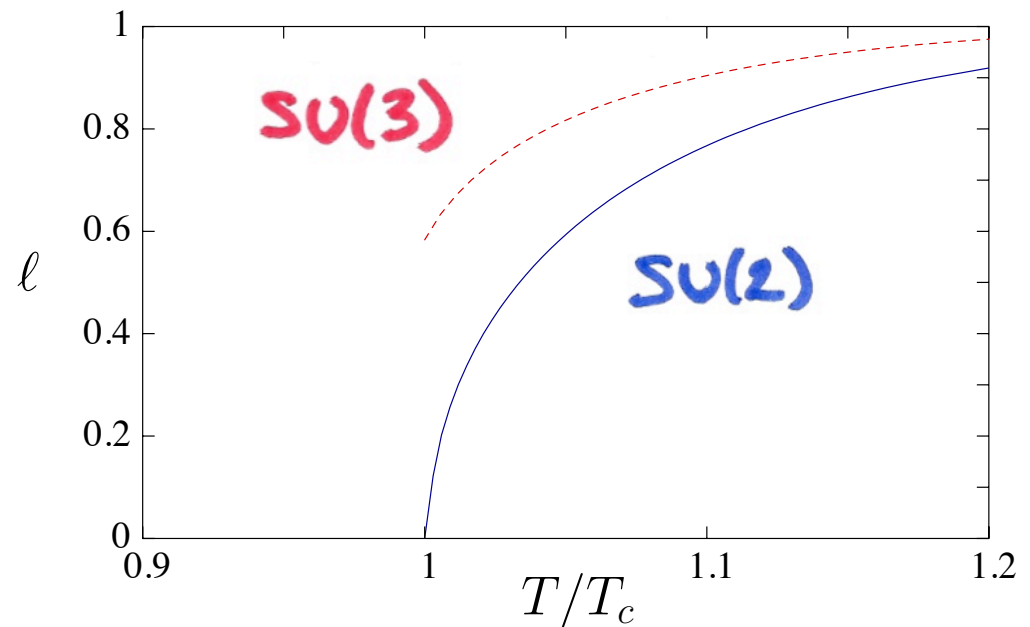
	$T_c/m$	$m$	$T_c$	$T_c^{\text{latt.}}$ *
SU(2)	0.33	710	238	295
SU(3)	0.36	510	185	270

(All in MeV)



## The Polyakov loop

$$l \sim e^{-F_9/T}$$

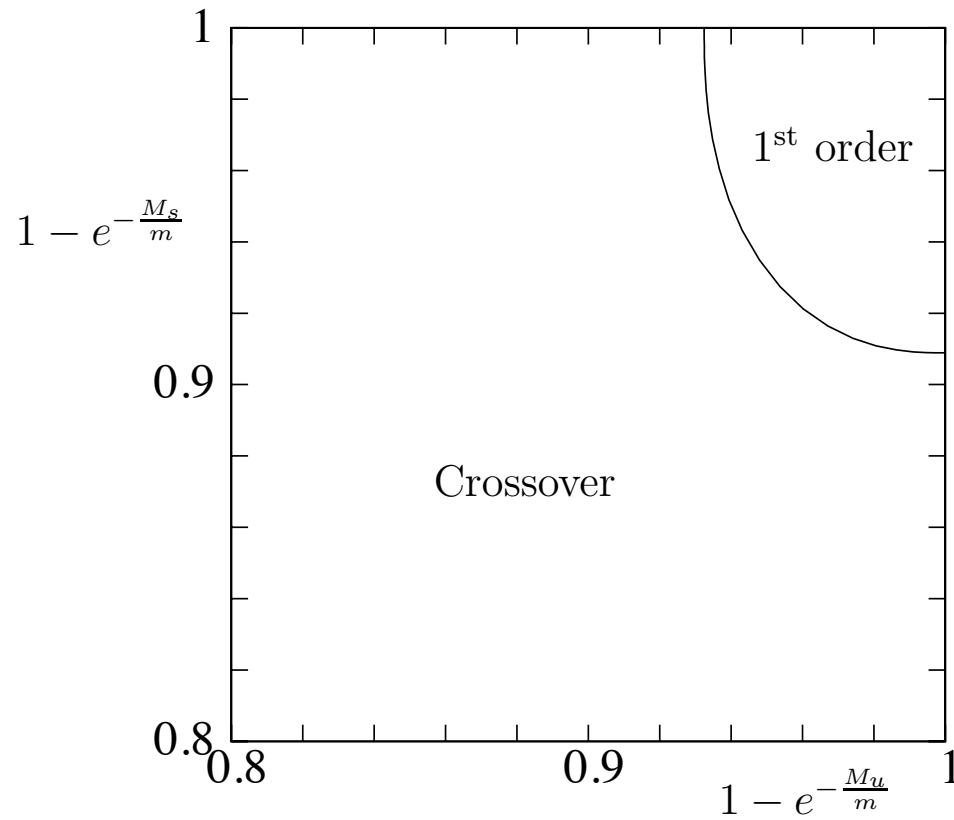


	$T_c/m$	$m$	$T_c$	$T_c^{\text{latt.}}$ *
SU(2)	0.33	710	238	295
SU(3)	0.36	510	185	270

(All in MeV)

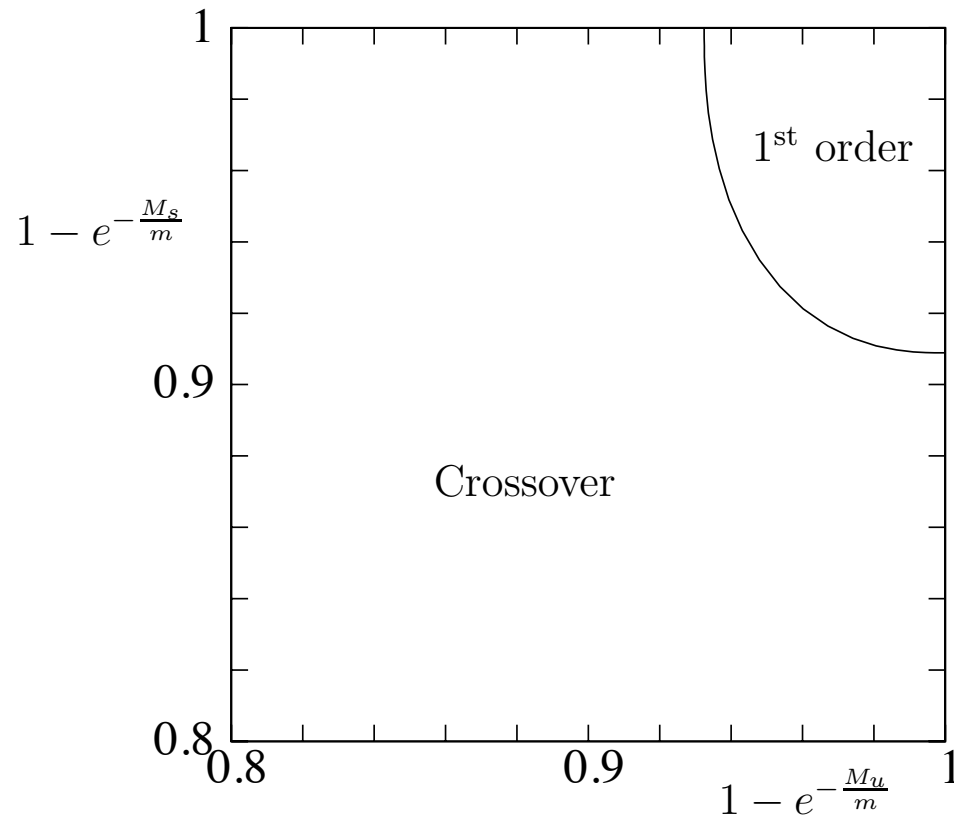
(de)confinement of static color charges from pert. theory!

# Adding (heavy) quarks ...



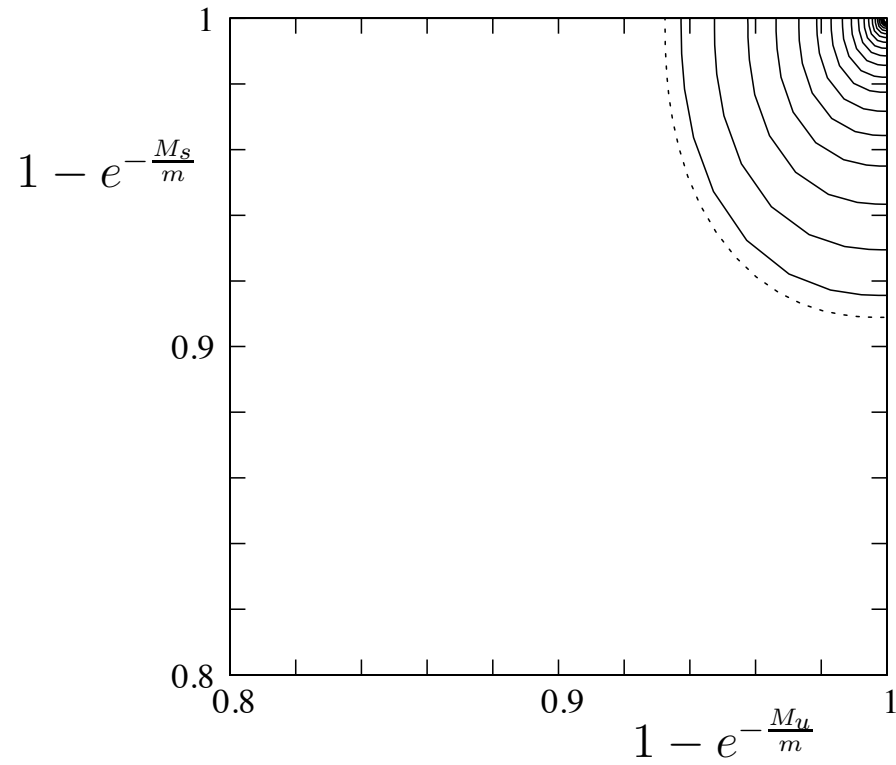


# Adding (heavy) quarks ...

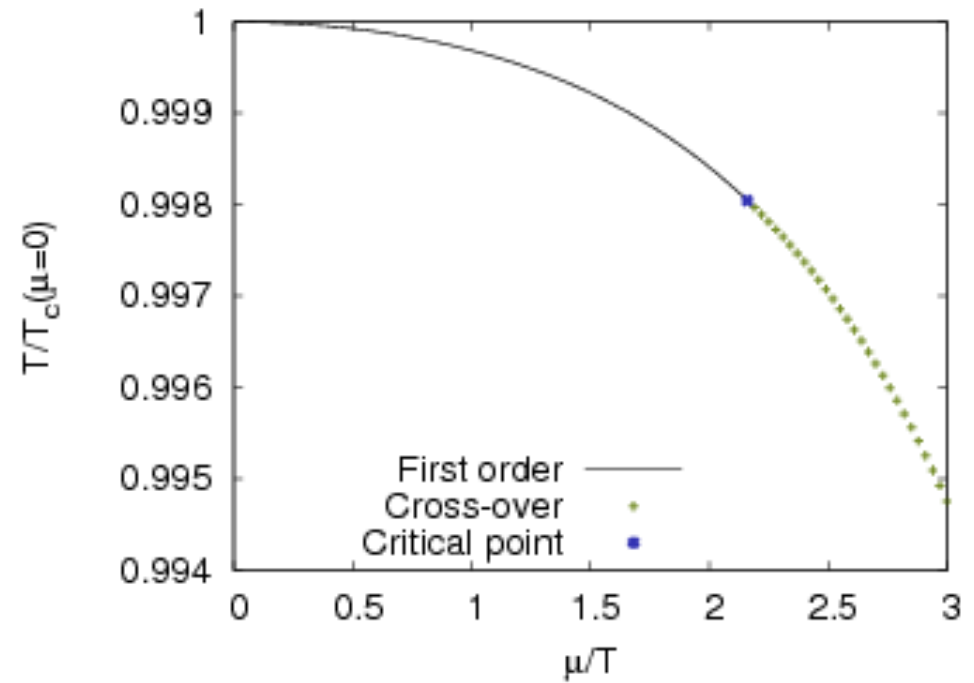
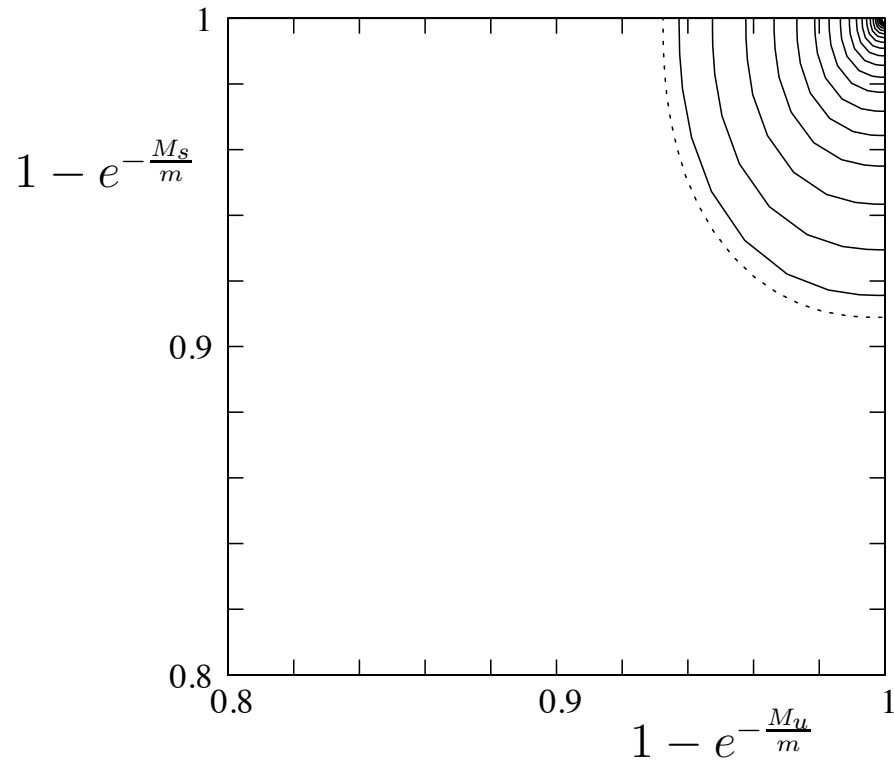


$N_f$	$M_c/T_c$	latt.*
1	<u>6.74</u>	7.22
2	<u>7.59</u>	7.91
3	<u>8.07</u>	8.32

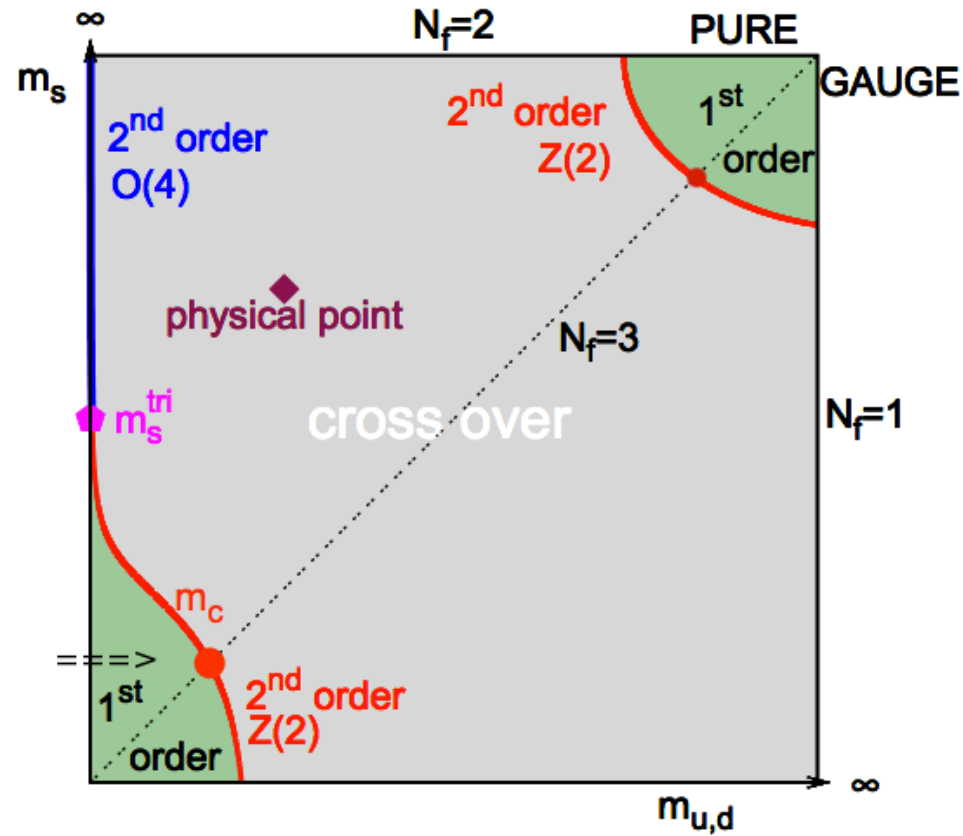
With a chemical potential



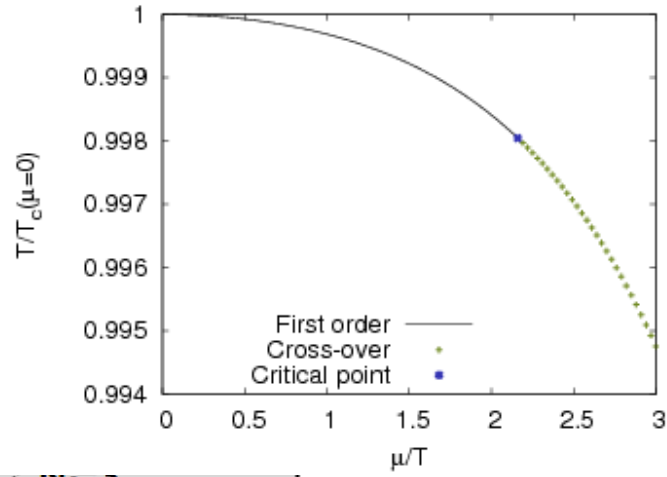
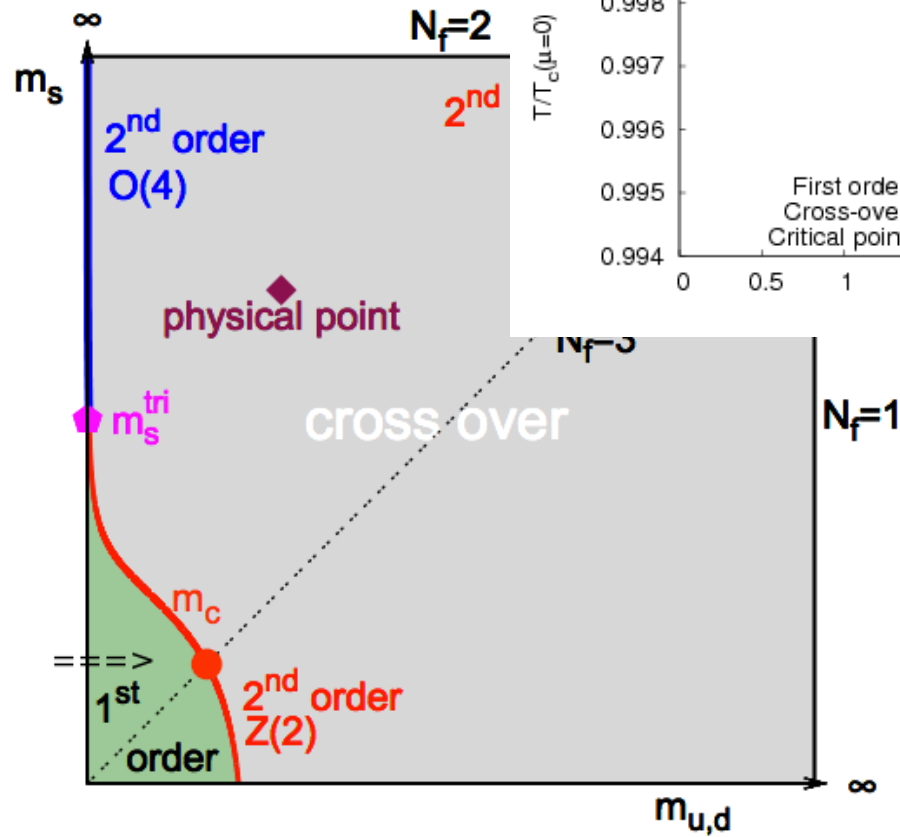
With a chemical potential



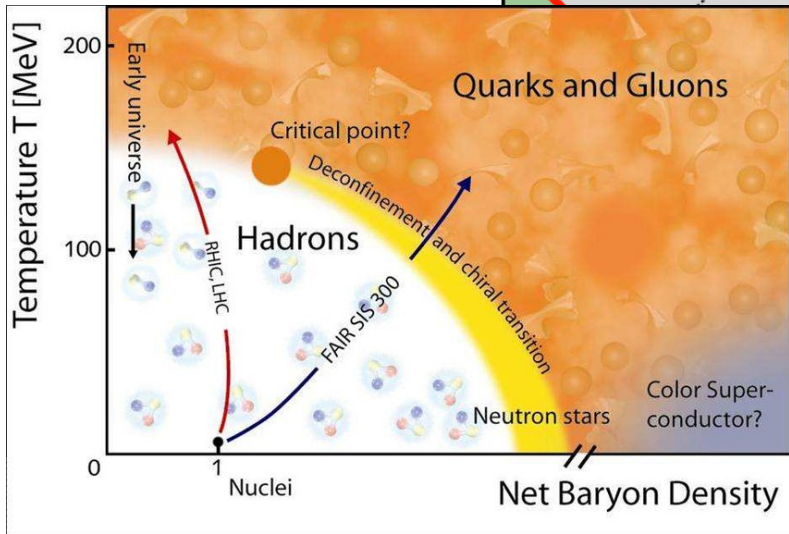
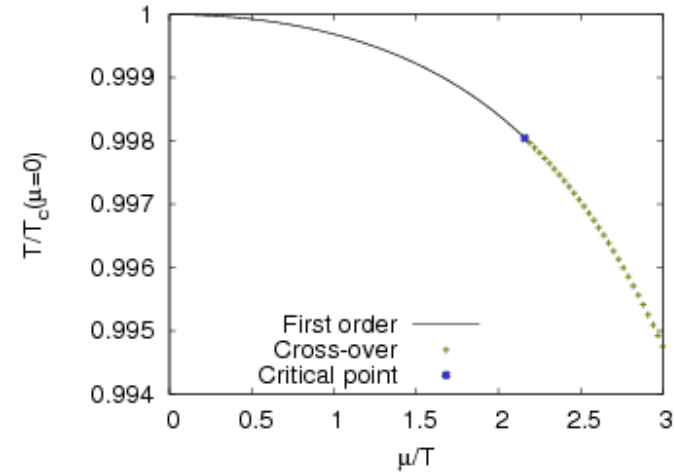
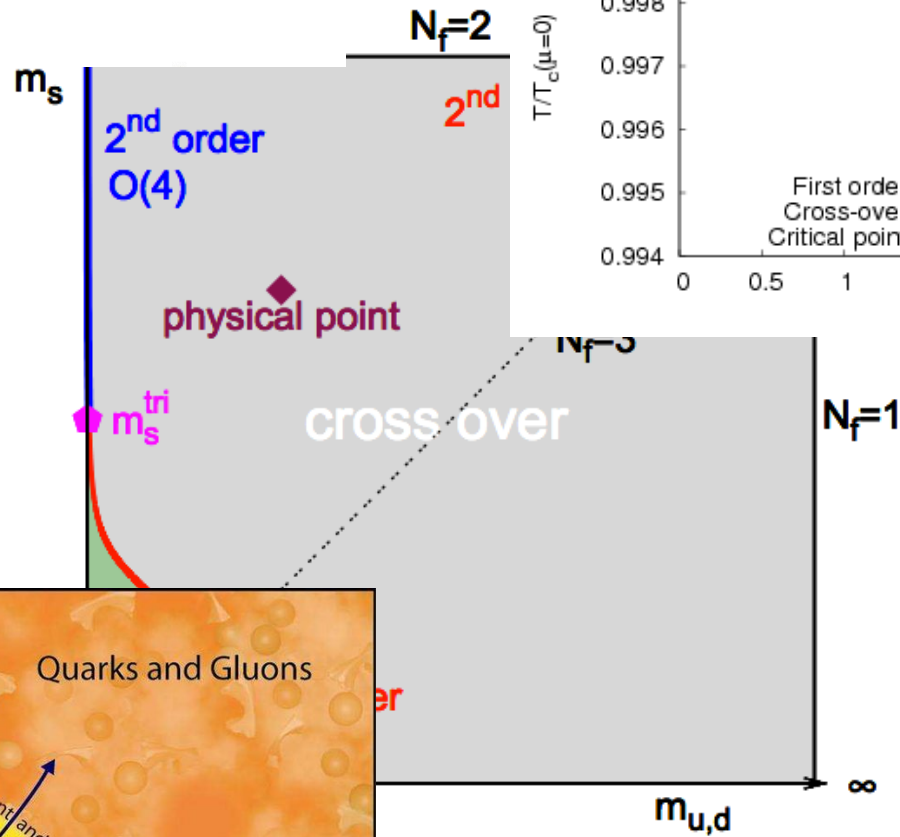
Light quarks ?



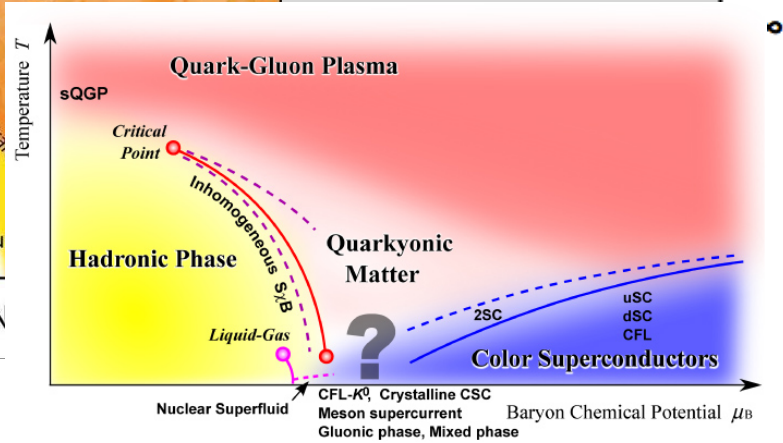
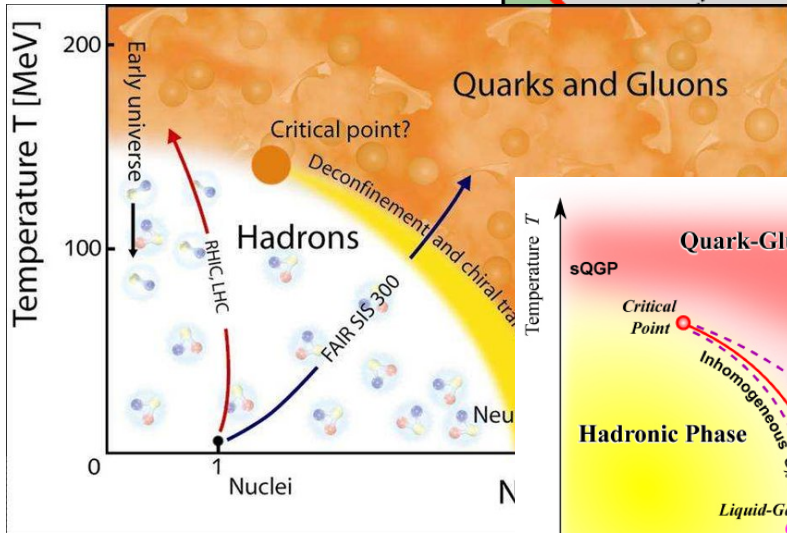
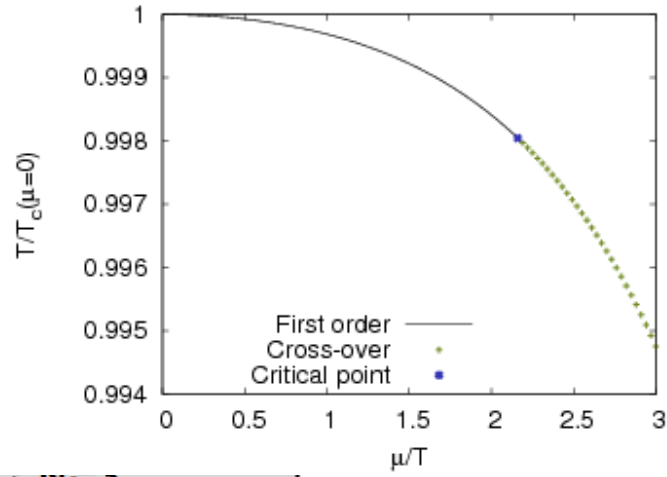
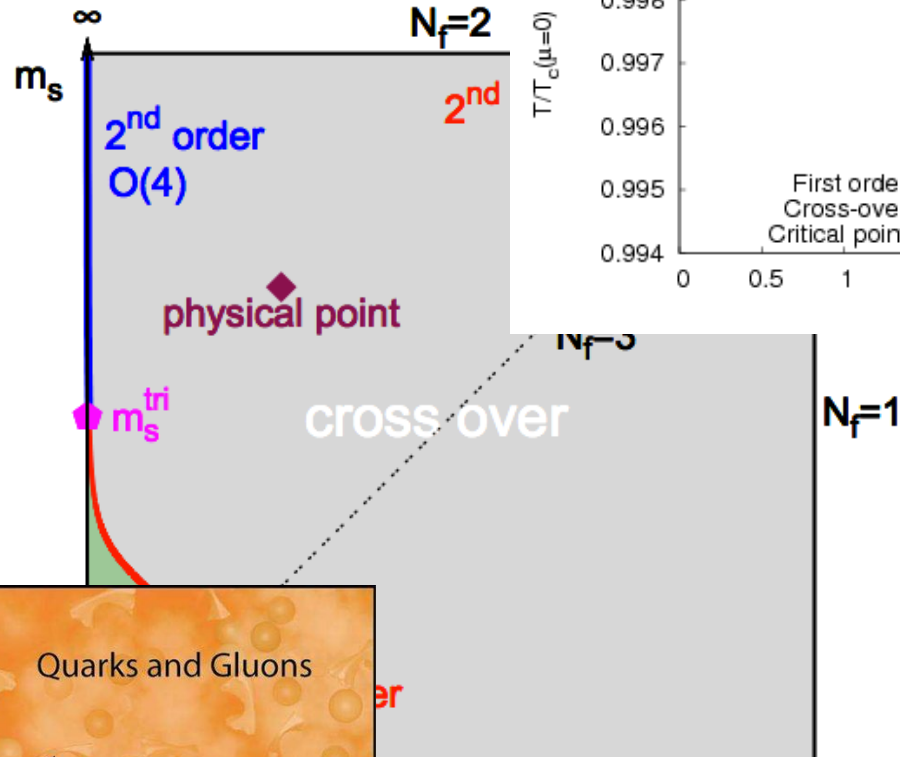
Light quarks ?



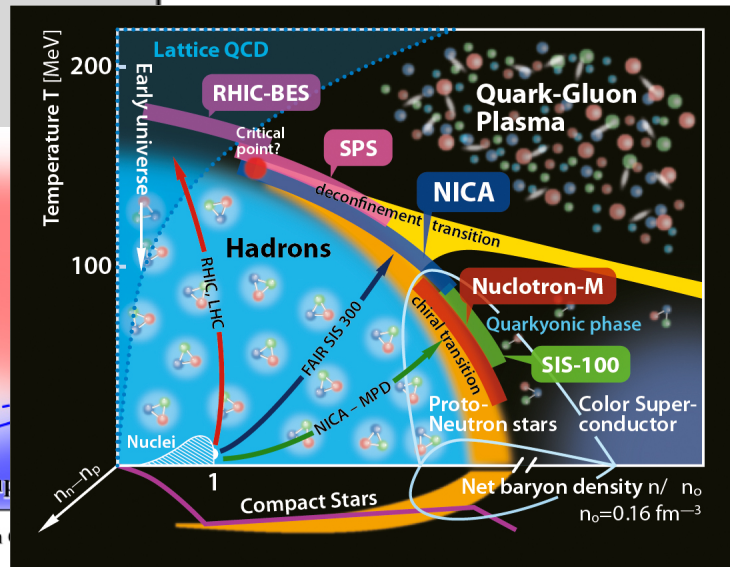
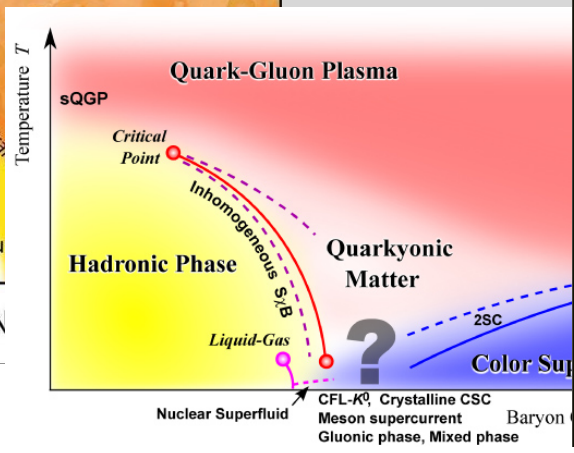
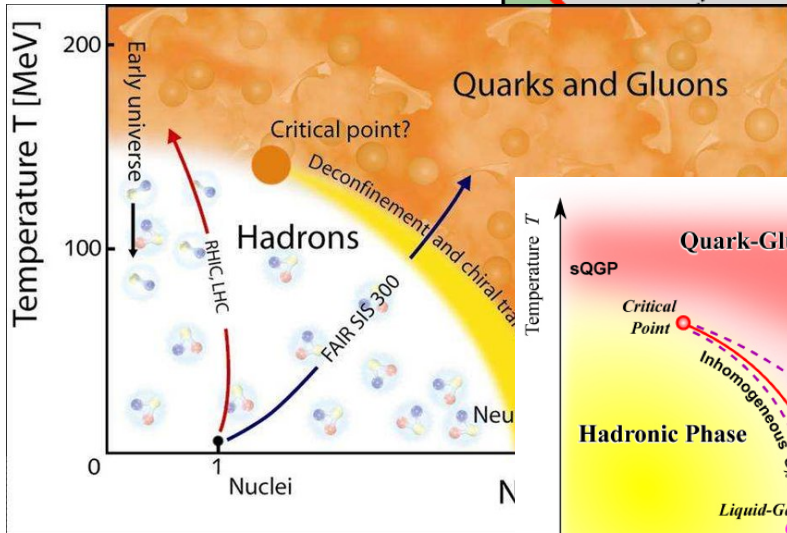
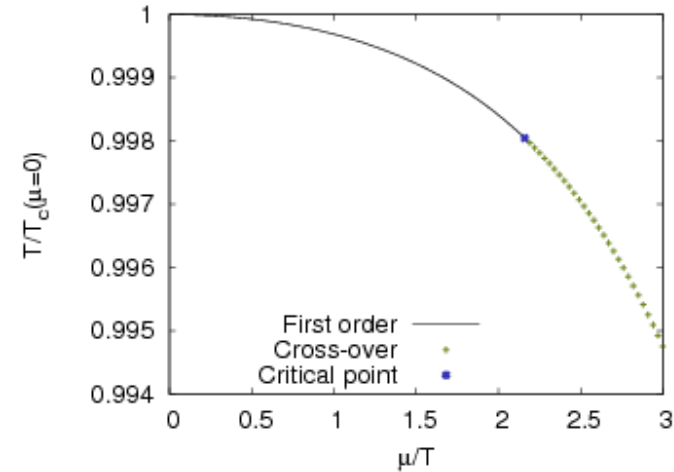
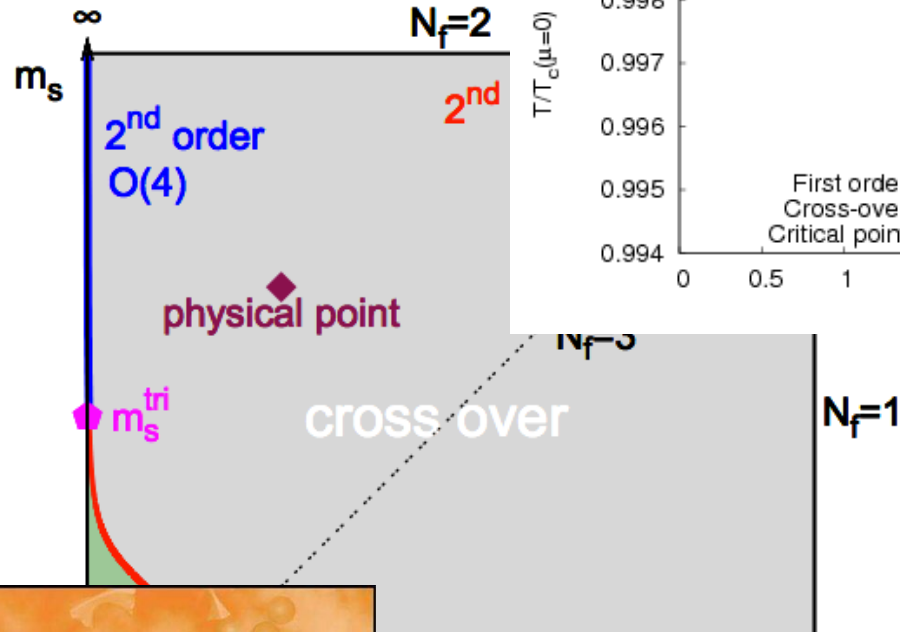
# Light quarks ?



# Light quarks ?

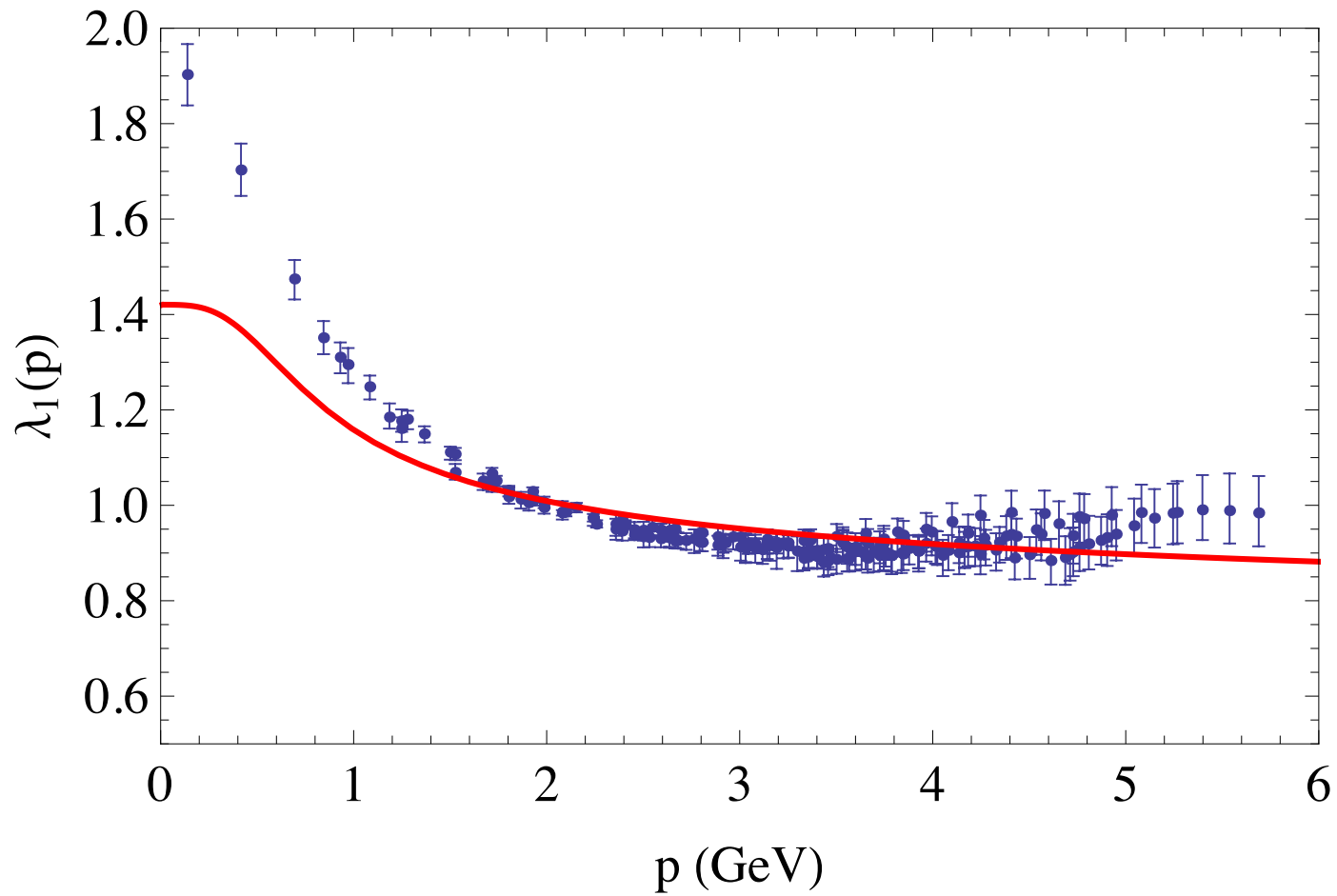


# Light quarks ?

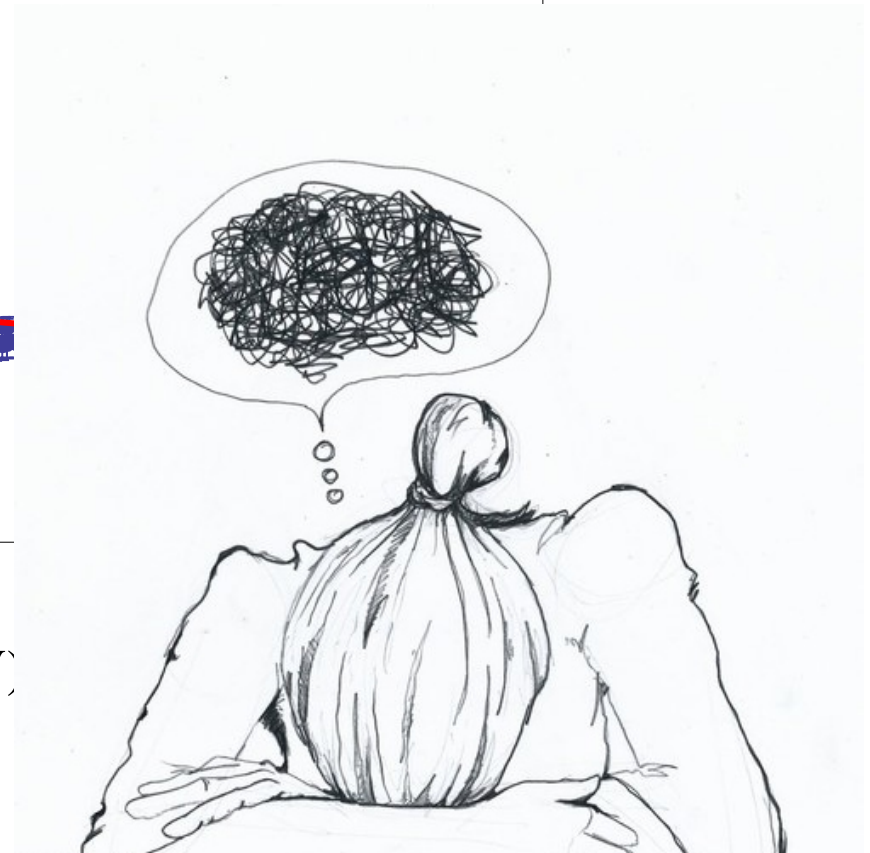
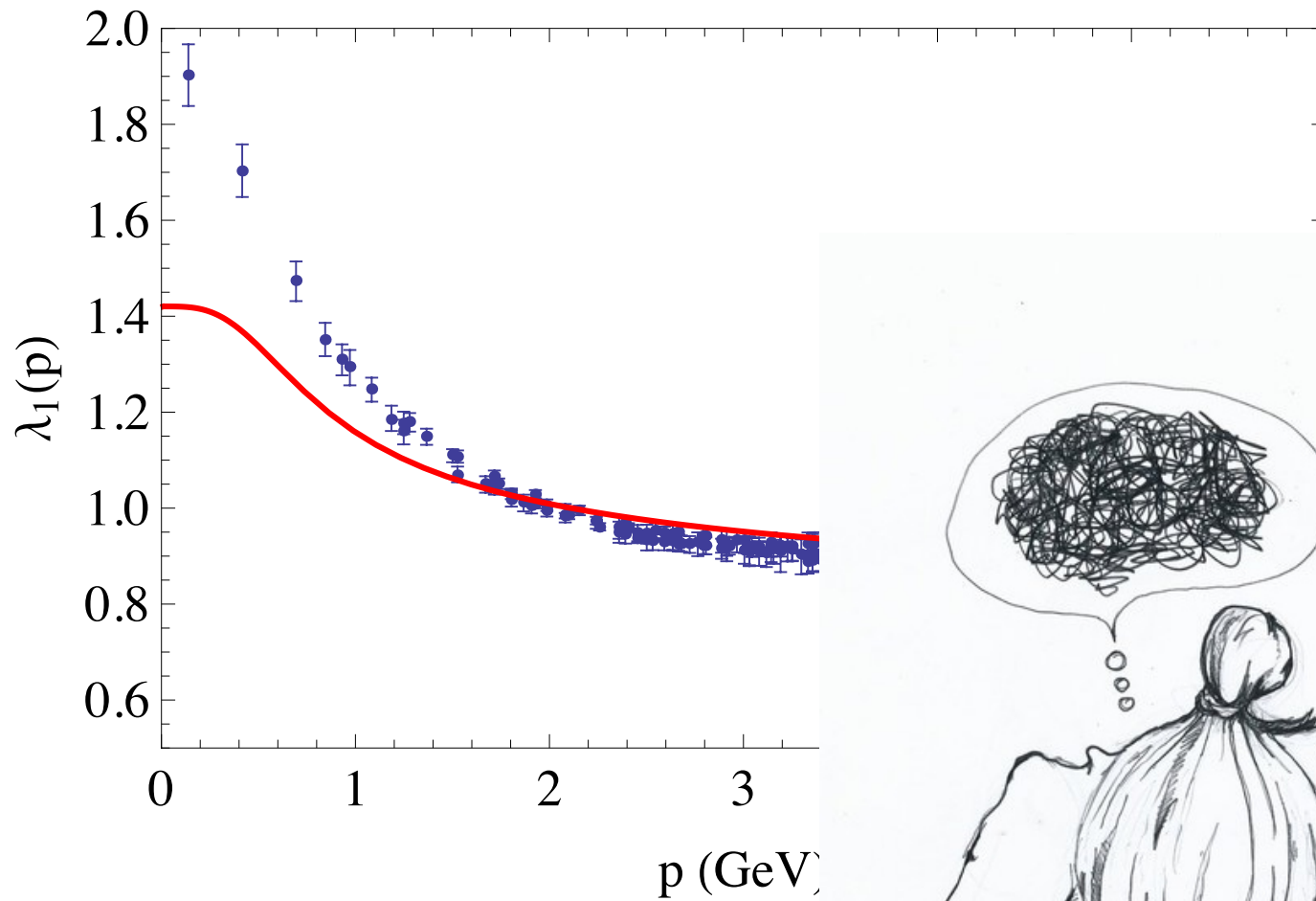




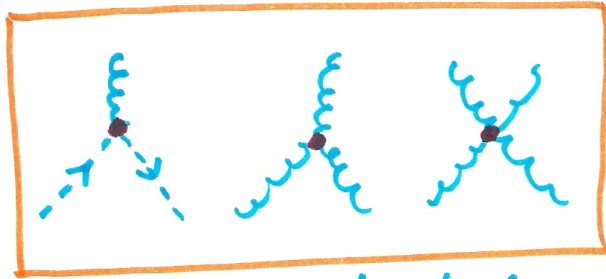
The quark-gluon coupling  
in the infrared



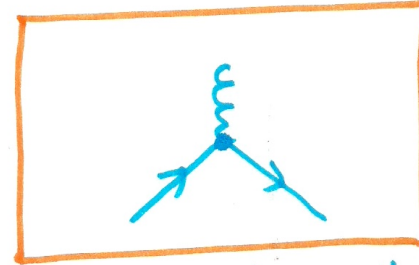
# The quark-gluon coupling in the infrared



# Small parameters in infrared QCD

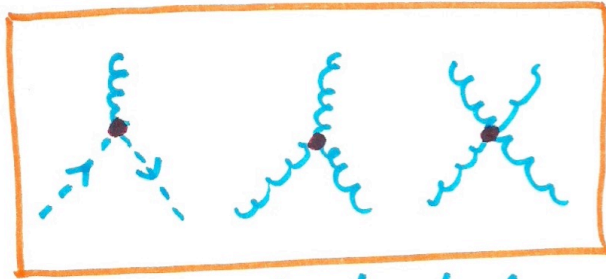


$g_g$ : can be treated perturbatively

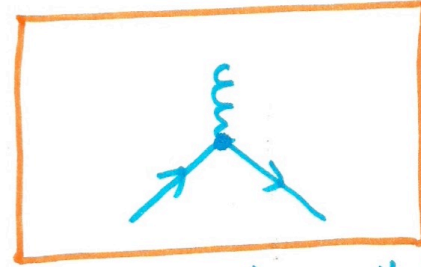


$g_q$ : not small !!

# Small parameters in infrared QCD



$g_s$ : can be treated perturbatively

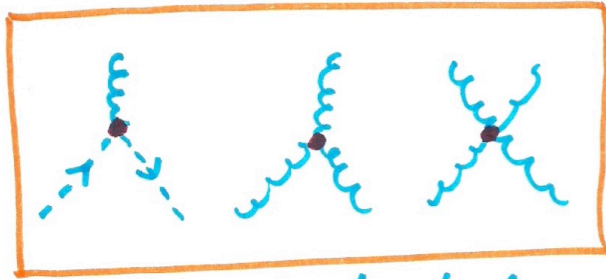


$g_q$ : not small !!

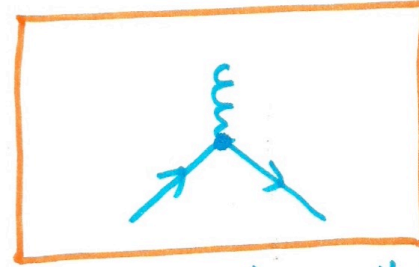
① Expand in  $g_s$ , not in  $g_q$

e.g. : only QED-like diagrams survive at L.O.  
in the quark sector

# Small parameters in infrared QCD



$g_g$ : can be treated perturbatively



$g_g$ : not small !!

① Expand in  $g_g$ , not in  $g_q$

e.g. : only QED-like diagrams survive at L.O. in the quark sector

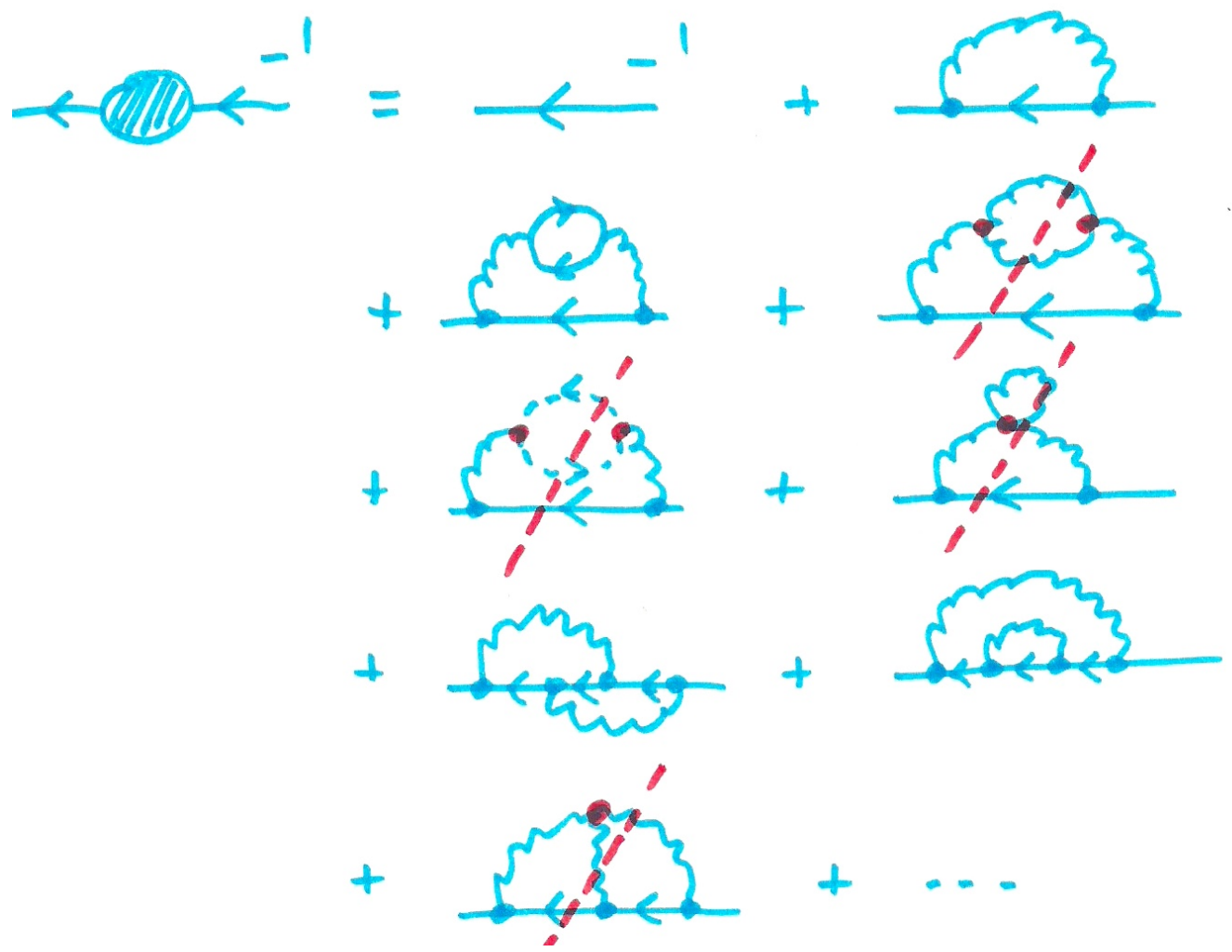
② Expand in  $1/N_c$  with  $g_q^2 N_c$  fixed  
[ 't Hooft ('74) ... ]

↙ Captures essential aspects of QCD dynamics

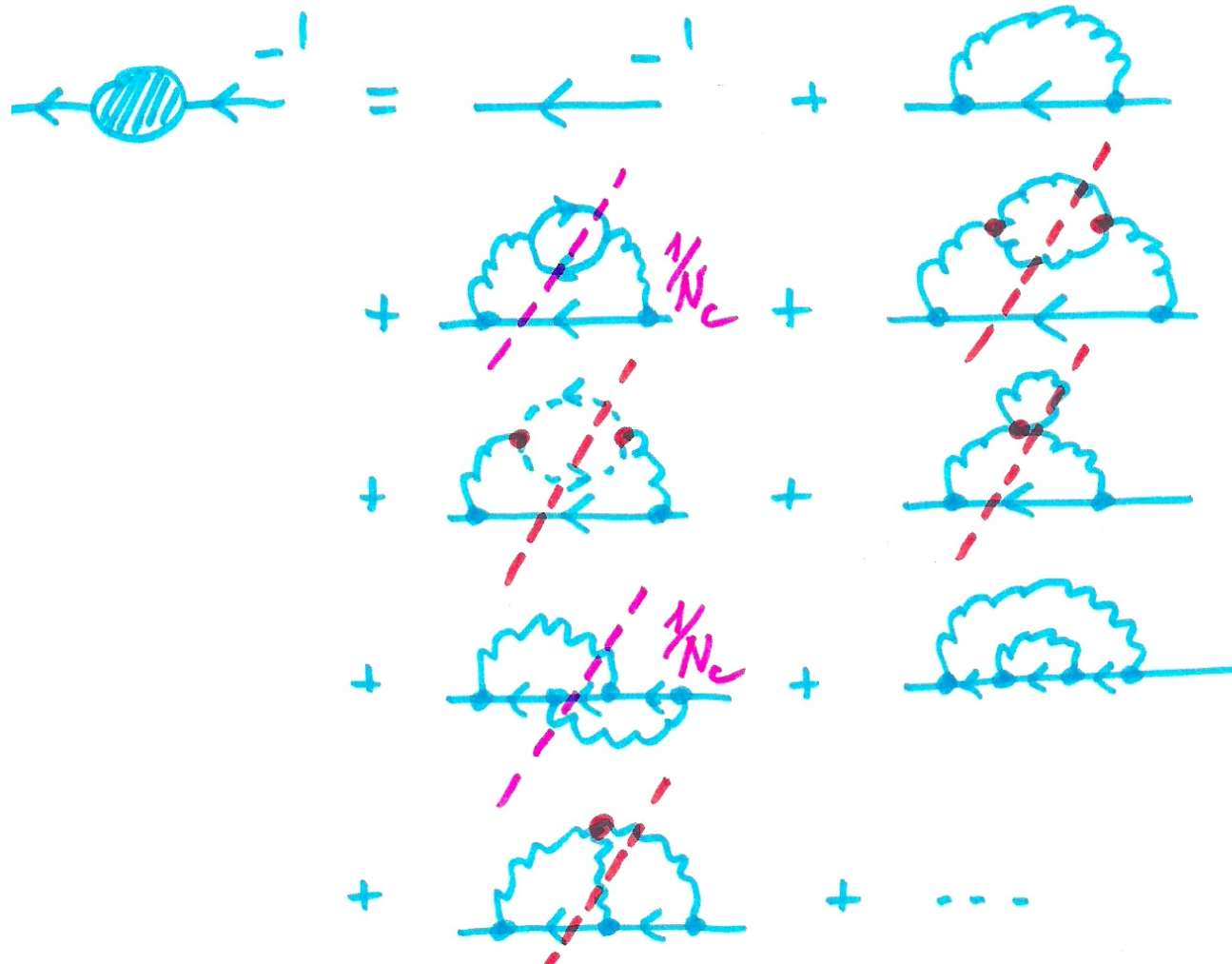
"Rainbow-improved" loop expansion



# "Rainbow-improved" loop expansion

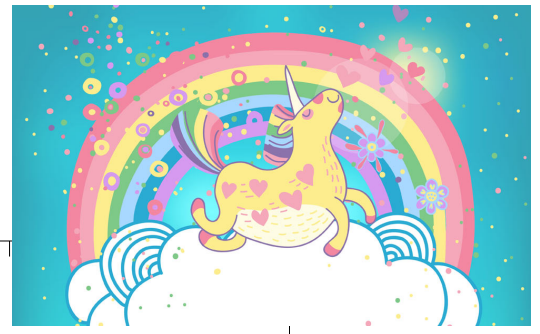
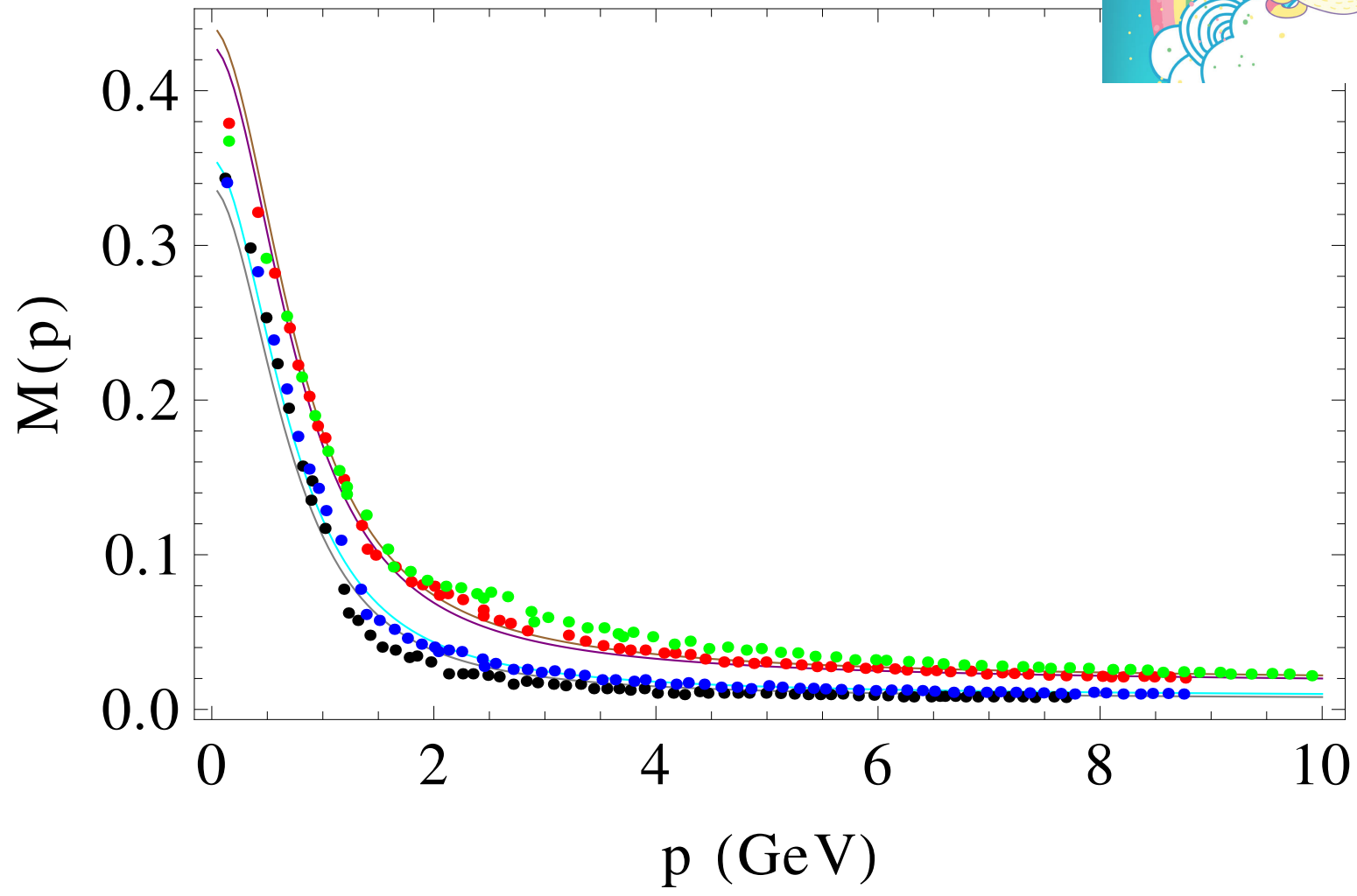


# "Rainbow-improved" loop expansion

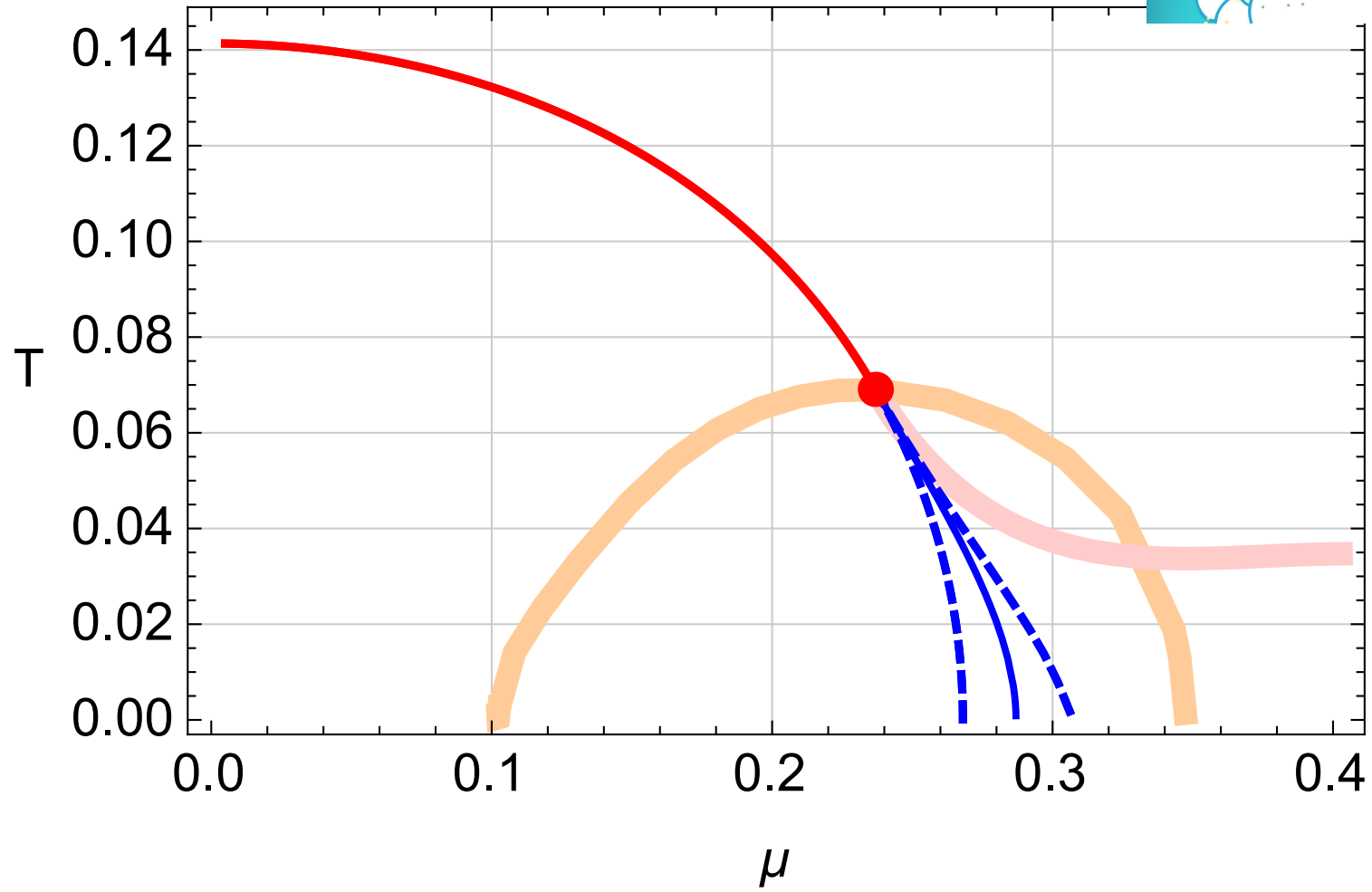






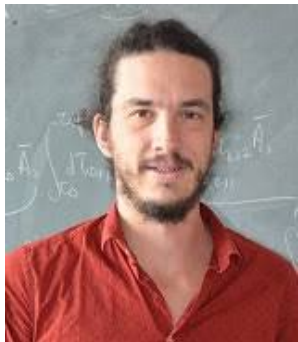


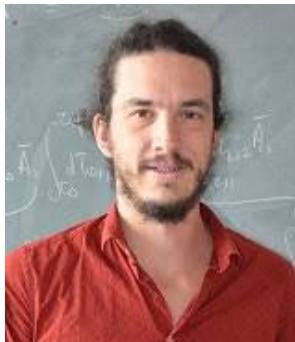


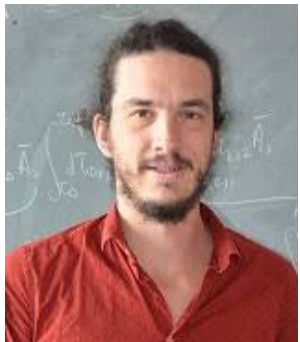
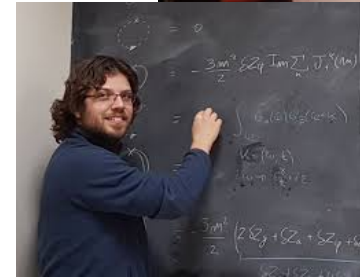


# THE TEAM

















$$Z \bar{z}_i(z) [(-1+S)S_{ue} + \beta^3] C_2^0(z)$$

$$\left[ \begin{aligned} & \frac{S \ln S}{\beta^3} + \frac{3M_+^4}{M_-^2 \Delta} \ln \frac{M_+^L}{\mu^L} - \frac{3M_+^2 S}{M_-^2 \Delta} \ln \frac{S}{\mu^L} \\ & - \frac{3M_-^4}{M_+^2 \Delta} \ln \frac{M_-^L}{\mu^L} + \frac{3M_-^2 S}{M_+^2 \Delta} \ln \frac{S}{\mu^L} \end{aligned} \right]$$



$$\sum_i \bar{c}_i(\alpha) [(-\Pi + S)S_{ice} + \beta S] C_i^0(\alpha)$$

$$= \frac{S \ln S/\mu^L}{\beta^3} + \frac{3M_+^4}{M_-^2 \Delta} \ln \frac{M_+^L}{\mu^L} - \frac{3M_+^2 S}{M_-^2 \Delta} \ln \frac{S}{\mu^L} + \frac{3M_-^2 S}{M_+^2 \Delta} \ln \frac{S}{\mu^L}$$

$$= + \frac{S}{\mu^L} \frac{S - \beta S}{\mu^L - S}$$

$$\left[ \frac{M_+^2}{\mu^L - S} (T_S - T_{M_+^2}) - \frac{M_-^2}{\mu^L - S} (T_S - T_{M_-^2}) \right]$$

$$\frac{Z}{g^{2/3}} = \frac{1}{16\pi^2 \epsilon} \left( 1 + \frac{3}{\Delta} (\Pi_+^2 - \Pi_-^2) \right) + \frac{1}{16\pi^2} \left[ \frac{S - \beta S}{\mu^L} \right]$$

$$= \frac{Z}{g^{2/3}} = \frac{Z}{g^{2/3}} = \frac{Z}{g^{2/3}}$$



$$\sum \bar{c}_i(x) [(-\pi + S) S_{ue} + \beta \beta] C_i^0(x)$$

$$\left[ \frac{S \ln S / \mu^L}{\beta^3} + \frac{3M_+^4}{M_-^2 \Delta} \ln \frac{M_+^L}{\mu^L} - \frac{3M_+^2 S}{M_-^2 \Delta} \ln \frac{S}{\mu^L} + \frac{3M_-^2 S}{M_+^2 \Delta} \ln \frac{S}{\mu^L} \right]$$

$$\left[ \frac{M_+^2}{M_-^2 - S} (T_S - T_{M_+^2}) - \frac{M_-^2}{M_+^2 - S} (T_S - T_{M_-^2}) \right]$$

$$\frac{Z}{g^{2/3}} = \frac{1}{16\pi^2 \epsilon} \left( 1 + \frac{3}{\Delta} (\pi_+^2 - \pi_-^2) \right) + \frac{1}{16\pi^2} \left[ \frac{S - \beta^3}{\beta^3} \right]$$

$$\left( T_S - T_{M_+^2} \right)$$

$$+ \frac{1}{16\pi^2} \left[ \frac{S - \beta^3}{\beta^3} \ln \frac{S - \beta^3}{\mu^L} - \frac{S \ln S / \mu^L}{\beta^3} + \frac{3M_+^4}{M_-^2 \Delta} \ln \frac{M_+^L}{\mu^L} - \frac{3M_+^2 S}{M_-^2 \Delta} \ln \frac{S}{\mu^L} + \frac{3M_-^2 S}{M_+^2 \Delta} \ln \frac{S}{\mu^L} \right]$$



$$Z \bar{C}_i(z) [(-1+S)S_{ue} + \beta^3] C_i^0(z)$$

$$\left[ \frac{S \ln S / \beta^3}{\beta^3} + \frac{3M_+^4}{M_-^2 \Delta} \ln \frac{M_+^L}{\beta^L} - \frac{3M_+^2 S}{M_-^2 \Delta} \ln \frac{S}{\beta^L} + \frac{3M_-^2 S}{M_+^2 \Delta} \ln \frac{S}{\beta^L} \right]$$

$$\left[ \frac{M_+^2}{M_-^2 - S} (T_S - T_{M_+^2}) - \frac{M_-^2}{M_+^2 - S} (T_S - T_{M_-^2}) \right]$$

$$\frac{Z}{g^{2/3}} = \frac{1}{16\pi^2 \epsilon} \left( 1 + \dots \right)$$

$$\left( T_S - T_{M_+^2} \right)$$

$$+ \frac{1}{16\pi^2} \left[ \frac{S - \beta^3}{\beta^3} \ln S - \frac{\beta^3}{\beta^L} - \frac{S \ln S / \beta^3}{\beta^3} + \frac{3M_+^4}{M_-^2 \Delta} \right]$$





$$\sum_i \bar{c}_i(z) [(-1+S)S_{ice} + \beta\beta] C_i^0(z)$$

$$\left[ \frac{S \ln S / \beta^2}{\beta^3} + \frac{3M_+^4}{M_-^2 \Delta} \ln \frac{M_+^L}{\beta^L} - \frac{3M_+^2 S}{M_-^2 \Delta} \ln \frac{S}{\beta^L} + \frac{3M_-^2 S}{M_+^2 \Delta} \ln \frac{S}{\beta^L} \right]$$

$$\left[ \frac{M_+^2}{M_-^2 - S} (T_S - T_{M_+^2}) - \frac{M_-^2}{M_+^2 - S} (T_S - T_{M_-^2}) \right]$$

$$\frac{z}{g^{2/3}} = \frac{1}{16\pi^2 \epsilon} \left( 1 + \dots \right)$$

$$\left( T_S - T_{M_+^2} \right)$$

$$+ \frac{1}{16\pi^2} \left[ \frac{S - \beta^3}{\beta^3} \ln S - \frac{\beta^3}{\beta^L} - \frac{S \ln S / \beta^L}{\beta^3} + \frac{3M_+^4}{M_-^2 \Delta} \right]$$



$$= + \frac{S}{\pi^2} \left( \frac{\pi_+^2}{4} - \beta S \right)$$


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$$\left[ \frac{M_+^2}{4 - S} \left( T_S - T_{M_+^2} \right) - \frac{M_-^2}{\pi^2 - S} \left( T_S - T_{M_-^2} \right) \right]$$

$$\frac{2}{g^2 \beta^3} = \frac{1}{16\pi^2 \epsilon} \left( 1 + \frac{3}{\Delta} \left( \pi_+^2 - \pi_-^2 \right) \right) + \frac{1}{16\pi^2} \left\{ \frac{S - \beta^2}{\beta} \right\}$$


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$$\underbrace{\left[ \frac{1}{16\pi^2 \epsilon} \left( 1 + \frac{3}{\Delta} \left( \pi_+^2 - \pi_-^2 \right) \right) \right]}_4 = \frac{2}{3\pi^2} - \frac{2}{34\pi^2}$$



$$\frac{d^3k}{(2\pi)^3} \theta(k - aH\epsilon) (a_{\vec{k}} f_{\vec{k}}(t) + a_{-\vec{k}}^+ f_{\vec{k}}^*(t)) e^{i\vec{k}\cdot\vec{x} - \omega t}$$

$${}_{+m} \psi \rangle = -\epsilon a H^2 (1) \\ = F(t, \vec{x})$$

$$\langle 2) F(t, \vec{x}') \rangle =$$

$$\frac{1}{2} m^2 A_{\mu}^a A_{\mu}^a$$

$$n_k = \mu_k = 0 \iff$$

$$N_k = 0 \quad \alpha_k = \alpha$$

$$1 + M S(t)$$

$$\frac{\pi}{a_4} = -V' + \frac{3}{4} \frac{\pi}{a_4} p \quad \dot{p} = \frac{\pi}{a_4}$$

$$\dot{\phi} = p + \frac{3}{4} p \\ \dot{p} = -V' + dH p + \frac{3}{4} p$$

$$\sim \frac{m^2}{H^2} \langle \frac{3}{4} p \rangle$$

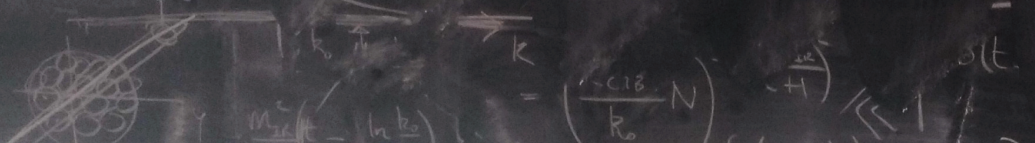
$$= \langle a_{\vec{k}}^+ a_{\vec{k}} \rangle$$

$$\mu_k = \langle a_{\vec{k}} a_{-\vec{k}} \rangle$$

$$f_{\vec{k}} = a_{\vec{k}} \mu_k + a_{-\vec{k}}^+ \mu_k$$

$$= b_{\vec{k}} f_{\vec{k}} + b_{-\vec{k}}^+ f_{\vec{k}}$$

$$\text{avec } f_{\vec{k}} = \alpha_k \mu_k +$$



$$\frac{d^3k}{(2\pi)^3} \theta(k - aH\epsilon) (a_{\vec{k}} f_{\vec{k}}(t) + a_{-\vec{k}}^{\dagger} f_{\vec{k}}^*(t)) e^{i\vec{k}\cdot\vec{x} - i\omega t}$$

$$\frac{\pi}{a_d} = -V' + \frac{3}{2} \frac{\pi}{a_d} p \quad \dot{p} = \frac{\pi}{a_d}$$

$$\dot{\phi} = p + \frac{3}{4} p^2$$

$$\dot{p} = -V' + dH p + \frac{3}{2} p^2$$

$$+m^2 \psi \rangle = -\epsilon a H^2 (1$$

$$= F(t, \vec{x})$$

$$\sim \frac{m^2}{H^2} \langle \psi \psi \rangle$$

$$\langle \psi(t, \vec{x}') \rangle =$$

$$\frac{1}{2} m^2 A_p^a A_p^a$$

$$= \langle a_{\vec{k}}^{\dagger} a_{\vec{k}} \rangle$$

$$\mu_k = \langle a_{\vec{k}} a_{-\vec{k}} \rangle$$

$$n_k = \mu_k = 0 \iff$$

$$N_k = 0 \quad n_k = \alpha$$

$$1 + M^2 S(t)$$

A mass out of the mess?



440 Toasts  
Carly  
Wish  
Day  
Holiday  
Friends  
Cake  
Happy  
Anniversary  
Birth  
Wish  
Present  
Age  
Surprise  
Happy  
Friends  
Holiday  
Party  
Anniversary  
Gifts  
Friends  
Day  
Wish



**I AM NOT**  
**61**  
**I AM 18**  
**WITH**  
**43** YEARS  
**OF**  
**EXPERIENCE**

440 Ted  
Candy  
Do  
Birth  
Wish  
A  
Sweet  
H

Birth  
Sary  
oy  
Friday  
ifts  
ends  
Present  
ay  
Wish  
Am Coming Betty