

Magnetically confined Fusion plasmas: Past, Present, Perspectives

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Outline

□ Magnetic confinement: an old story

Some recent developments at CPHT

□ What next?

Fusion: a sun in a box, really?





Proton-proton chain reaction

An excessively slow reaction

Gravitationnel confinement



150 t E ~200 W.m⁻³

- 15 x 10⁶ K
- 150 g.cm⁻³



Easier (and dirtier) reactions



 ${}^{2}_{1}D + {}^{2}_{1}D \longrightarrow {}^{3}_{1}T + p^{+} + 4.0 MeV$ $\longrightarrow {}^{3}_{2}He + n^{0} + 3.3 MeV$ $\Box \text{ Without neutron emission (aneutronic)}:$ ${}^{2}_{1}D + {}^{3}_{2}He \longrightarrow {}^{4}_{2}He + p^{+} + 18.3 MeV$ ${}^{3}_{2}He + {}^{3}_{2}He \longrightarrow {}^{4}_{2}He + 2p^{+} + 12.9 MeV$

 $p + {}^{11}_{5}B \longrightarrow 3{}^{4}_{2}He + 8.7MeV$



50 keV ~ 600 Millions K

□ The plasma is heated by the alpha particles

$$P_{\alpha} = \Delta E_f \langle \sigma v \rangle n_e^2 V / 20$$

□ It undergoes losses (conduction, radiation, etc.)

$$P_{\rm loss} = W_{\rm th} / \tau_E \qquad W_{\rm th} = 3n_e kT$$

 \Box Losses have to be compensated by α -particle heating

Lawson criterion
$$n_e \tau_E = \frac{60T}{\langle \sigma v \rangle \Delta E_f}$$

Ignition criterion tells you how good you are



- Inertial fusion: If you don't want to bother confining your plasma for a long time, you'll have to increase the density significantly
- Magnetically confined fusion: If you don't like the idea of having solid like densities in your plasmas, then you'll have to confine your plasma over macroscopic times

Magnetic confinement

- **D** Plasma density $n_e \sim 10^{19} 10^{20} \text{m}^{-3}$
- □ Necessary confinement time :

$$\tau_E \sim 0.1 - 1 \mathrm{s}$$





Higher field means smaller Larmor radius: particles are confined in 2 out of 3 directions

- \Box Particles undergo a mirror force $-\mu \nabla_{\parallel} B$
- Too many particles escape through the edges

The torus solution conundrum



The culprits are drifts



More generally for any transverse force, a drift

$$\boldsymbol{v}_D = \frac{1}{q} \frac{\mathbf{F} \times \boldsymbol{B}}{B^2}$$

A helical field becomes necessary



The origin of current instabilities





- Flexible current coil in a magnetic field
- □ Ideal instability can develop
- □ Characteristic time : Alfven time

$$\frac{d^2 a}{dt^2} = \frac{n}{\tau_A^2} a$$
$$\tau_A = \sqrt{\frac{R\rho S}{IB_0}}$$

The sawtooth instability [Von Goeler 1974]



Fast ejection of the core in less than 100μs

After a relaxation period (10ms – 1s), the process starts again

Wide diversity of phenomenological behaviours



How do we study this problem?

NUCLEAR FUSION 5 (1965)

NECESSITY OF THE ENERGY PRINCIPLES FOR MAGNETOSTATIC STABILITY

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Three energy principles for magnetostatic stability are known that are supposed to give necessary and sufficient conditions. For this reason their minimization has been the subject of a lot of work in plasma physics. Indeed one can easily justify the sufficiency of the conditions of stability devised from these energy principles. But to the demonstrate their necessity one usually assumes that the operators of the perturbed linearized motions have a complete spectrum of eigenfunctions in the space of square integrable functions. We show the weakness of that assumption and propose two new demonstrations of the necessity of energy principles for stability that require less stringent assumptions. The first demonstration involves some properties of the Laplace transform. In the second one we use the integral invariants of the linearized motion. The conclusions in both cases are identical: if one finds a trial function which makes the potential energy negative, the equilibrium is unstable. We give lower and upper bounds for the growth rate of the unstable perturbation.

$$\rho \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = \delta \mathbf{J} \times \mathbf{B} + \mathbf{J} \times \delta \mathbf{B} - \nabla \delta p \equiv \mathbf{F}(\boldsymbol{\xi})$$

$$\delta W = \frac{1}{2} \int d^3 \boldsymbol{x} \Big[|\delta \mathbf{B}_{\perp}|^2 + \frac{B^2}{\mu_0} |\nabla \cdot \boldsymbol{\xi}_{\perp} + 2\boldsymbol{\xi}_{\perp} \cdot \boldsymbol{\kappa}|^2 + \Gamma p |\nabla \cdot \boldsymbol{\xi}|^2 \quad \begin{array}{c} \text{Current} \\ \text{term} \\ -(\boldsymbol{\xi}_{\perp} \cdot \nabla p)(2\boldsymbol{\xi}_{\perp}^{\star} \cdot \boldsymbol{\kappa}) - J_{\parallel}(\boldsymbol{\xi}_{\perp} \times \mathbf{B}) \cdot \delta \mathbf{B}_{\perp} \Big]$$

How do we study this problem?

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Internal Kink Modes in Toroidal Plasmas with Circular Cross Sections

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The stability criterion of the internal kink mode is given in toroidal geometry for plasmas with circular cross sections. Contrary to known results in cylindrical geometry, the internal kink mode can be stable if the pressure gradient is sufficiently low.

 $\gamma \propto \left(\beta_p^2 - \beta_{pc}^2\right)$

 β_p represents the pressure content within the instability range

The structure of an internal kink



The structure of an internal kink



First nonlinear simulations (cylindrical)

NUCLEAR FUSION, Vol.31, No.11 (1991)

TRANSITION FROM A RESISTIVE KINK MODE TO KADOMTSEV RECONNECTION

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ABSTRACT. The scaling of the non-linear reconnection process associated with the m = 1 internal kink instability is studied in cylindrical geometry, using a three-dimensional numerical code with a full set of resistive MHD equations. In the presence of an ideal unstable kink mode, the non-linear evolution of the instability shows a transition from a purely resistive kink mode for high resistivity to Kadomtsev reconnection driven by the ideal kink mode for low resistivity. For the case of low resistivity, the assumptions of the Kadomtsev reconnection model have been checked, and the results confirm Kadomtsev's estimations of a scaling law of $\eta^{1/2}$. A model is proposed to understand the transition and to compare the studies with previous numerical results obtained for different plasma parameters.



FIG. 2. Displacement of the magnetic axis versus time (in Alfvén time) for values of η ranging from 10^{-4} to 10^{-7} . The displacement $\xi_0 = 0.15$ a, corresponding to the saturation of the ideal kink, is indicated by the dotted line.

Cylindrical simulations: paving the way for the XTOR code

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The equations solved by XTOR-2F code

[Lütjens & Luciani 2008, 2010]

Continuity equation

 $\partial_t \rho = -\nabla \cdot (\rho \boldsymbol{v}) + \mathbf{D} \nabla^2 \rho$

Pressure equation

$$\partial_t p = -\boldsymbol{v} \cdot \nabla p - \Gamma p \nabla \cdot \boldsymbol{v} + \boldsymbol{\chi}_\perp \nabla^2 p$$

Induction equation

 $\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \boldsymbol{\eta} \nabla^2 \boldsymbol{B}$

 $\partial_t \boldsymbol{v} = \frac{1}{\rho} \left(\boldsymbol{J} \times \boldsymbol{B} - \nabla p \right) - \boldsymbol{v} \cdot \nabla \boldsymbol{v} + \boldsymbol{\nu} \nabla^2 \boldsymbol{v}$

More complete 2-fluid model

$$\begin{aligned} \frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{v} + n\boldsymbol{v}_{i}^{\star}) &= \nabla \cdot D_{\perp} \nabla n + S_{n} \\ n \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} + \boldsymbol{v}_{i}^{\star} \cdot \nabla \boldsymbol{v}_{\perp} \right) &= \boldsymbol{J} \times \boldsymbol{B} - \nabla p + \nabla \left(\nu \nabla \left(\boldsymbol{v} + \boldsymbol{v}_{i}^{\star} \right) \right) \\ \frac{\partial p}{\partial t} + \boldsymbol{v} \cdot \nabla p + \Gamma p \nabla \cdot \boldsymbol{v} + \frac{2}{3} \nabla \cdot \mathbf{q}^{\star} &= \nabla \cdot \left(n \chi_{\perp} \nabla \frac{p}{n} \right) + \nabla \cdot \left(n \chi_{\parallel} \nabla_{\parallel} \frac{p}{n} \right) + H \\ \frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times \left(\boldsymbol{v} \times \boldsymbol{B} \right) + \frac{1}{\omega_{c,i} \tau_{A}} \nabla \times \left(\frac{\nabla_{\parallel} p_{e}}{n} \boldsymbol{b} \right) - \nabla \times \left(\eta \boldsymbol{J} \right) \end{aligned}$$

□ The diamagnetic (ion in red, electron in blue), account for:

- Rotation of the mode and the plasma
- Linear stabilisation of internal kinks
- □ Strong nonlinear acceleration effects

Succesful simulations of Sawteeth and Tearing





- First sustained sawtooth cycles [Halpern PoP 2011]
- Fast crash using electron diamagnetic effects

- Modelling of density dynamics during the crash, well compared to experimental observations [Nicolas PoP 2012]
- Impurity dynamics [Nicolas PoP 2013]

Succesful simulations of Sawteeth and Tearing



Outline

- □ Focus at CPHT on the internal kink mode for decades
- Sophisticated tools developed and leading research carried out
- Sawtooth cycles and a few puzzling phenomena elucidated
- □ Long-lasting problems still unsolved (fast crash)
- A new playground opens with the hybrid kinetic/MHD code XTOR-K

Kinetic effects



□ Particle motion: 3 periodic motions

Q Resonance with fluid modes for the slowest precession motion

Can be stabilizing or destabilizing

Sawteeth as a root cause for disruption



The hybrid approach to model energetic particle effects

$$\mathbf{\Pi}_{k} = \int_{-\infty}^{+\infty} \mathbf{u}_{k} \otimes \mathbf{u}_{k} f_{k}(\mathbf{u}, \mathbf{r}, t) d^{3}\mathbf{u} = \sum_{i=1}^{N_{k}} \mathbf{u}_{k,i} \otimes \mathbf{u}_{k,i} \delta(\mathbf{r} - \mathbf{r}_{k,i}(t))$$



- Particles are integrated in 6D (Boris-Buneman)
- **Typically 100 M particles**
- Nonlinear phase not accessible yet
- Typically 2-3 Picard iterations are necessary

The hybrid approach to model energetic particle effects



Physics of the fast crash: difficult





- □ First plasmoids obtained with XTOR-2F
- But unrealistic if one wants to compute relevant resolutions
- □ Not sure how to go further here

Conclusion

□ Still a lot to say about