# Perturbation Theory for Soft Pion Physics?

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## Introduction and Plan of Talk

I present here first the contribution of Roland Seneor to the Vietnamese Higher Education. As you all known, Roland passed away a few years ago. I then give a brief review of the Particle Physics group in its first 30 years. The remaining time is to discuss various non perturbative methods such as the solution of the Muskelishvili -Omns Integral equation, the Pad Approximant and the Inverse Amplitude as applied to a simple problem of the Pion Vector Form Factor where there are plenty of experimental data to check the non perturbative methods. This non perturbative approach, where unitarity is respected, is in constrast with the modern Chiral Perturbation Theory of Weinberg.

## Contribution of Roland Seneor to Higher Education in Viet Nam

Let me first present to you the contribution of Roland Seneor. Under his direction a large group of foreign students came to Ecole Polytechnique to study and to interact with the traditional French students which is an important part of the education program at Ecole Polytechnique. Because of the historical relation, the first important group which was recruited to arrive here was from Vietnam. I gave Roland a helping hand during his reign as the Director of the  Direction des Relations Extrieurs  During this period Roland has brought to France a group of the best students from Viet Nam. To the surprise of many people one of them became a major at Polytechnique. Some of them have returned to Vietnam and have played an important rle in education and in industry. One of them has become the Vice Chancelor of the French Vietnamese Technical University in Hanoi. Those who decided to stay abroad have occupied important positions at well known universities in France as at Science Po. University Paris XIII and in Europe as Ecole Polytechnique de Lausanne ... Roland contribution was recognized by the vietnamese ministry of education by awarding a joint award medal to him and myself (photo)


Figure: Ceremony of receiving the Medal of "Contribution to Higher Education " given to Roland Seneor and TNT" by the Ministry of Education

Figure: Roland Seneor and TNT were invited by the General Giap to have tea at his residence in order to show his appreciation for our effort and also for the generosity of the Ecole Polytechnique

Although I gave Roland a helping hand in recruiting the vietnamese students, I must say my effort was not complete without the helping hand of my wife Thanh Thuy who made an important effort of inviting them to our house and treating them with vietnamese food and thus provided them a smooth transition to the French life.

## A Brief Review of the Particle Physics Group

The scientific work of the phenomenological group dealt with properties of quark and leptons. Perturbative and nonperturbative approach were used in the study. Low energy properties of the Quantum Chromo Dynamics were studied. Quark models as one aspect of QCD were studied, Chiral Symmetry of the pion as a Nambu-Goldstone bosons using the current algebra technique for the study of the Tau Lepton semi hadronic decays as well as the threshold multiple pion profuction in electron positron collision. The neutrino oscillation phenomena was also studied as well as the properties of the B meson decays. A large effort was dedicated to the study of the phenomenology of the short distance behavior of the QCD which is undoubtedly the theory of the strong interaction: the parton distribution function, the deeply virtual Compton scattering. Numerical calculation of the lattice QCD was undertaken.

Georges Grunberg did by himself two spectacular papers in 1980 and 1982 on the Renormalization Group Improved Perturbative QCD and Renormalisation Scheme Invariant QCD and QED which have more than 500 citations. It is extremely rare to see such highly quoted publications written by one person. When I understood the importance of his 2 papers, I did ask influential people a promotion for him but it was not granted. This was an injustice.

The arrival of I. Antoniadis to the Particle Physics group brought happily the future of this group in the direction of the particle physics beyond the Standard Model. He has deveveloped a group of String Theory which eventually splitted from the Particle Physics group.

T. N. Pham was nominated in 2013 by Phys. Rev. and Phys. Rev. Letters Editors as an Outstanding Reviewer. The same honor was awarded this year to Urko Reinosa.

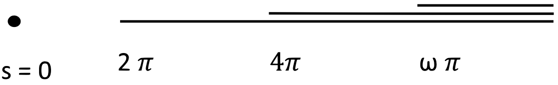
## A Brief Review of Weinberg Current Algebra Calculation

Current Algebra (CA) by Weinberg in the late 1960's using the LSZ reduction formula was a landmark in this field of Physics. All his CA results can be easily derived by Effective Lagrangian method taking into account of the fundamental properties of the

Pion Goldstone Boson. The development of the physics based on the Effective Lagrangian is known as the Chiral Perturbation Theory. Just the same as in the Weinberg CA approach, the unitarity question was neglected in the perturbative approach.
In the early 1981 I pointed out that the discrepancy between the Weinberg calculation and experimental data of the process $K_{l4} \to \pi\pi e\nu$ is due to the neglect of the pion final state interaction as required by the unitarity constraint. Later that year Roiesnel and myself pointed out that the Weinberg calculation for the process$\eta \to \pi^+\pi^-\pi^0$ needed a large correction due to the neglect of the final state interaction. These two works showed the the final di pion interaction cannot be neglected. They are complicated to be presented here. In the following the Vector Pion form factor calculation is explicitly shown to satisfy the elastic unitarity relation with the inelastic correction taken into account in a manner which is consistent with analyticiy.

## Analytic properties of the Vector Pion Form Factor

Analytic Properties of the Pion Form Factor F(s)
in the Complex s-Plane




$$F(s) = \sum_n c_n s^n$$

Figure: Below the inelastic threshold, the inelastic contribution to the form factor can be expanded in a power series in s

## Calculation of the Vector Pion Form Factor

### The Muskelshvilli Omnes Integral Equation for the Vector Pion Form Factor

Using the elastic unitarity approximation for the form factor V(s), one arrives at the Muskheshivilli Omnnes Integral equation:

$$V(s) = 1 + \frac{s}{s_{R_1}} + \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{V(z) f_1^*(z)}{z^2(z - s - i\epsilon)} dz \tag{1}$$

The solution of the integral equation is well known:

$$V(s) = P_n(s) D^{-1}(s) \tag{2}$$

where

$$D^{-1}(s) = \exp(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_1(z)}{z(z-s-i\epsilon)}dz) \tag{3}$$

The polynomial $P_n(s)$ is the ambiguity of the solution of the MO equation; it is normalised $P_n(0) = 1$ due to the Ward identity. The physical interpretation of this term is mysterious. As we see from the analytic property of the form factor below the inelastic threshold, the contribution of the inelastic effect can be expanded in a power series in s. The inelastic effect can be expressed as a sum or a product a two cuts(as in the N/D method). The product solution is given by the solution of the MO integral equation which satisfies the final state theorem.

**a) Theoretical Consideration: One Loop CPTH Result**

The one loop perturbation graph is given in Fig.2

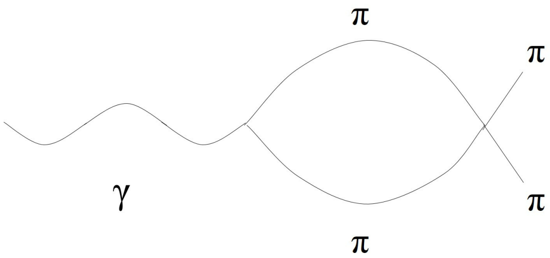



Figure: one loop perturbation calculation

The one loop ChPT result can be expressed in the dispersion language:

$$V^{pert.}(s) = 1 + \frac{s}{s_{R_1}} + \frac{1}{96\pi^2 f_\pi^2}((s - 4m_\pi^2)H_{\pi\pi}(s) + \frac{2s}{3}) \tag{4}$$

where $H_{\pi\pi}(s)$ is a well-known logarithm function and the last term behaves as $s^2$ at low s. The second term on the R.H.S. is the subtraction constant and is the experimental r.m.s. radius of the pion. The last term is the prediction of the one loop ChPT. For 2-loop calculation, one would need one more subtraction which can be determined by fitting data and the prediction of ChPT is the term which behaves at low s as $s^3$.

*The perturbative one-loop result is also the first iteration of the the MO integral solution. One can show the solution of the MO equation corresponds to an infinite iteration procedure.This shows that the ChPT perturbative result is unreliable.*

## Non Perturbative Result using the Pad Approximant method

Starting from the one loop result one can use either the inverse amplitude method or the Pad approximant method to get a non-perturbative result which satisfies the final state theorem.
Here we use the Pad approximant method:
Let f(z) be an analytic function in z and be defined by the Taylor series:

$$f(z) = \sum_{n=0}^{\infty} a_n z^n \tag{5}$$

The Pad approximant $f^{(N,M)(z)}$ of $f(z)$ is defined by the following rational fraction:

$$f^{(N,M)}(z) = \frac{P_N(z)}{Q_M(z)} = f(z) + O(z^{N+M+1}) \tag{6}$$

where $P_N(z)$ and $Q_N(z)$ are polynomial of degree N and M
We want to generalize this Pad approximant for the (complex) perturbative one loop amplitude.
The Pad [0,1] approximant reads (T. N. Truong, Phys Rev. Lett. 61, 2526 (1988)):

$$V^{(0,1)} = \frac{V^{tree}}{1 - \frac{V^{1-loop}}{V^{tree}}} \tag{7}$$

where $V^{tree}$ refers to the tree amplitude which is equal to unity and $V^{1-loop}$ refers to the one loop amplitude, i.e. the last two terms on the R.H.S. of Eq. (4). The Padé approximant method yields V(s) satisfying the elastic unitarity relation. It is just a bubble summation of the one-loop graph. Fixing the value of

$s_{R_1}$such that the phase of the form factor is 90 degree at the $\rho$ mass, the r.m.s. radius of the pion is 15 percent too small. *This is understandable because we neglect the inelastic and/or higher resonances contribution.* Below and sufficiently far from $s_i$, e.g. the $\omega\pi$ threshold, as it was explained above, we can parametrise the inelastic contribution by a polynomial $P_n(s)$ which is real for $s < s_i$ and is the polynomial ambiguity in the solution of the Muskhelishvilli-Omnès equation. We shall fit the experimental data below with the simple expression $(1 + \alpha s/s_\rho)$, where $\alpha$ is a constant to simulate the inelastic effect:

$$V^f(s) = \frac{1 + \alpha s/s_\rho}{1 - s/s_{R_1} - \frac{1}{96\pi^2 f_\pi^2}\{(s - 4m_\pi^2)H_{\pi\pi}(s) + 2s/3\}} \qquad (8)$$

*There are 2 parameters in this equation, $\alpha$ and $\rho$ mass. $\alpha$ which represents the inelastic effect can be fixed by the magnitude value of the pion form factor at the mass of $\rho$. The $\rho$ width is calculated and is simply the KSRF relation*. (Tran N. Truong Phys. Rev. D30, 1509 (1984))

One can unitarized the two-loop ChPT result for the pion form factor, using the inverse amplitude or the Pad approximant method to get a similar results in perfect agreement with the results presented above (T. Hannah Phys. Rev. D55,5613 (1997)). Hannah produced a series of paper to incorporate the unarity constraint in the ChPT calculations. Unfortunately his work was not known or recognized by ChPT workers.

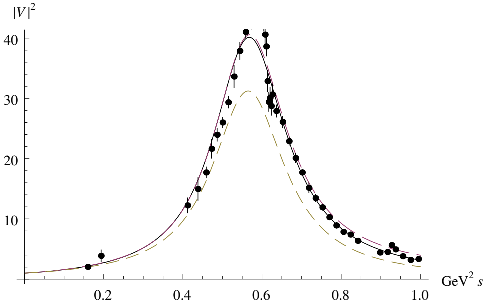



Figure: The square of the modulus of the vector pion form factor $V(s)$ as a function of the energy squared $s$. The experimental data are from $e^+e^-$ data. . The dash line curve is the IAM or Pade result (UChPT) of the one loop ChPT fitted to give the correct experimental P-wave phase shifts which goes through 90 degrees at the $\rho$ mass. It yields however a too low value of the r.m.s. pion radius. The solid curve is the UChPT multiplied by a nomial $(1{+}0.14\mathrm{s}/s_\rho)$ simulating the contribution from higher resonances and /or inelastic effects. The pion r.m.s. radius is now correct

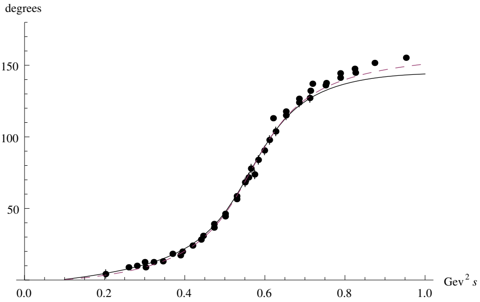



Figure: The solid circles are the experimental phase shifts as determined by the pion pion scattering, the solid curve is the calculated pion form factor phase as given by the UChPT calculation.

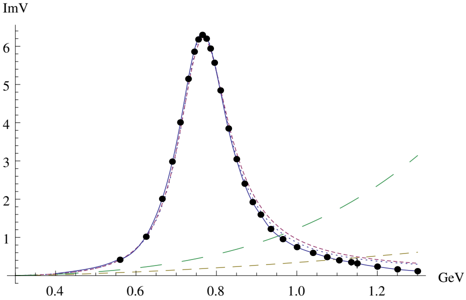



Figure: The imaginary part of the vector pion form factor $ImV(s)$ as a function of the energy in the $GeV$ unit. The solid curve is the experimental result as calculated by the elastic unitarity relation using the experimental data; the long-dashed curve is the two-loop ChPT calculation, the medium long-dashed curve is the one-loop ChPT calculation, the short-dashed curve is the UChPT one loop calculation with the inelastic effect and/or higher resonances taken into account.

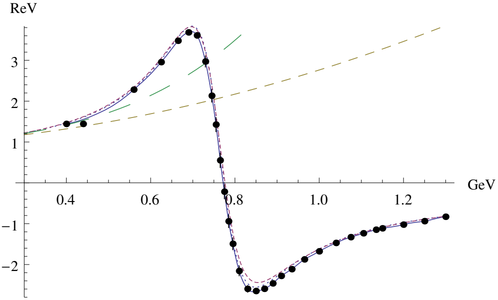



Figure: The real part of the vector pion form factor $ImV(s)$ as a function of the energy in the $GeV$ unit. The curves are the same as given in Fig. 3. Because the ReV(s) and ImV(s) are related by Dispersion Relation one must test agreement of V(s) with experimental data over a wige range of energy

## Conclusion

Unitarity or conservation of probability is the law of nature. You get into trouble if you violate it. It is amusing to note that ChPT was invented by Li and Pagels. Pagel was a former student of Lehmann. He went to America and published the paper on ChPT. Lehmann quickly wrote a paper to remind his student that it was allright to do the perturbation for chiral theory, but one has to unitarise the result where it was needed in order to respect unitarity. ...".Unfortunately Lehmann gave the title of his paper as an "Effective Range Expansion..." which could be misunderstood.The word Effective Range meant simply the unitarity constraint!

I would like to thank R. Blankenbecler, J. Lascoux (deceased) and M. Peshkin for their encouragement and support.